CS81 Assignment 2
Due Tuesday, 2 February 2010

1. Suppose that \( \Gamma \) and \( \Gamma' \) are sets of propositional formulas. For each statement below, if the statement is true, prove it. If it is false, give a counter-example.

   a. If \( \Gamma \) is satisfiable and \( \Gamma \subseteq \Delta \), then \( \Delta \) is satisfiable.

   b. If \( \Gamma \) is satisfiable and \( \Delta \subseteq \Gamma \), then \( \Delta \) is satisfiable.

   c. If \( \Gamma \) is satisfiable and \( \psi \) is a tautology, then \( \Gamma \cup \{\psi\} \) is satisfiable.

2. Show that \( \Gamma \cup \{\varphi\} \models \psi \) iff \( \Gamma \models \varphi \rightarrow \psi \) (used in the completeness theorem).

3. Prove the part of the induction step of the soundness theorem, as discussed in the lecture slides for Tuesday 26 January, for the case of the \( \lor \)Elimination rule.

4. For the tautology \( (\neg p \rightarrow p) \rightarrow p \), show the proof that would be constructed in the structural induction part of the proof of the completeness theorem given in the lecture slides.