

**CS81 Assignment 3**  
Tableau and Sequent Calculus  
Due Tuesday, 9 February 2010

You may check your work with automated tools and edit with JAPE, but please make sure you can do the problems unaided.

Using the tableau method, determine whether or not the sequents represent valid entailments. For any that are not, derive a valuation that falsifies the entailment based on the tableau.

1.  $\vdash (P \rightarrow Q) \vee (Q \rightarrow P)$
2.  $\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$
3.  $(P \rightarrow Q), (R \rightarrow S) \vdash (P \vee R) \rightarrow (Q \wedge S)$
4.  $\vdash (R \rightarrow S) \rightarrow ((P \rightarrow R) \rightarrow (P \rightarrow S))$
5.  $(P \rightarrow Q) \wedge (Q \rightarrow R), \neg R \vdash \neg P$

For 5, show it as a block tableau rather than as a tree. For the others, you may use either block or tree tableaux.

Prove, using the sequent calculus, or give a counterexample if the sequent is not provable.

6.  $P \rightarrow Q, \neg Q \vdash \neg P$
7.  $\neg(P \wedge \neg Q) \vdash \neg P \vee Q$
8.  $\vdash (R \rightarrow S) \rightarrow ((P \rightarrow R) \rightarrow (P \rightarrow S))$
9.  $(P \rightarrow Q) \wedge (Q \rightarrow R), \neg R \vdash \neg P$
10.  $\vdash ((P \rightarrow Q) \vee ((Q \rightarrow R) \wedge (R \rightarrow S)))$

For 8, give the corresponding block tableau (starting with the negation of the formula).