

**CS81 Assignment 4**  
Due Tuesday, 16 February 2010

Prove sequents 1-6 by natural deduction. Do each one by hand, to make sure you understand the process. You may use JAPE for guidance and formatting, either before or after doing it by hand. Turn in either your own construction, a JAPE screenshot, or a transcription from JAPE.

Please submit in single-sheet groupings as follows:

{1, 2}, {3, 4}, {5}, {6}, {7, 8}.

1. [5 points]       $\exists x(A(x) \rightarrow B(x)) \mid\text{---} (\forall xA(x)) \rightarrow (\exists xB(x))$
2. [10 points]     $\exists x\neg R(x) \mid\text{---} \neg\forall yR(y)$
3. [10 points]     $\exists x\neg R(x) \mid\text{---} \exists y(R(y) \rightarrow \forall yR(y))$
4. [15 points]     $\neg\forall x\neg R(x) \mid\text{---} \exists yR(y)$
5. [20 points]     $(\forall xA(x)) \rightarrow \exists xB(x) \mid\text{---} \exists x(A(x) \rightarrow B(x))$
6. [15 points]     $\exists xT \mid\text{---} \exists x(A(x) \rightarrow \forall yA(y))$

(Here T is “top” i.e. “true”, so  $\exists xT$  just means “something exists”.)

7. [5 points] Demonstrate an interpretation  $(\Delta, \mu)$  that shows the following formula is not valid:

$$\forall x\exists yR(x, y) \rightarrow \exists y\forall xR(x, y)$$

8. [20 points] Show that the following formula is true for every interpretation. (Give an informal proof from first principles that does not use natural deduction rules.)

$$\forall x(A(x) \vee B(x)) \rightarrow (\forall xA(x)) \vee (\exists xB(x))$$