CS81 Assignment 5
Due Tuesday, 23 February 2010

In 1-7, check each syllogism or sequent for validity by the tableau method. If not valid, give a counterexample. Assume that “a lover” means a person who loves someone.

1. Given
   a. Everyone loves a lover.
   b. Mary loves herself.
   It follows that
   c. John loves Mary.

2. Given
   a. Everyone loves a lover.
   b. Mary loves John.
   It follows that
   c. John loves Mary.

3. Given
   a. Everyone loves a lover.
   b. Mary loves herself.
   It follows that
   c. Mary loves John.

4. Given
   a. John loves everyone who does not love him/herself (and possibly others).
   It follows that
   b. John loves himself.

5. \( (\forall x \ A(x)) \rightarrow (\exists x \ B(x)) \rightarrow (\forall x \ (A(x)\rightarrow B(x))) \)

6. \( (\forall x \ A(x)) \rightarrow (\exists x \ B(x))) \mid — (\exists x \ (A(x)\rightarrow B(x))) \)

7. \( (\exists x \ (A(x)\rightarrow B(x))) \mid — ((\forall x \ A(x)) \rightarrow (\exists x \ B(x))) \)

In 8-10, prove by the Sequent Calculus, or give a counterexample.

8. \( (\forall x \ (A(x)\rightarrow B(x))) \mid — ((\forall x \ A(x)) \rightarrow (\exists x \ B(x))) \)

9. \( (\exists x \ (A(x)\rightarrow B(x))) \mid — ((\forall x \ A(x)) \rightarrow (\exists x \ B(x))) \)

10. \( (\forall x A(x)) \rightarrow \exists x B(x) \mid — \exists x(A(x) \rightarrow B(x)) \)