

## Computer Science 81, Spring 2010

### Assignment 10

Due Tue. April 13

Please use the following page grouping: (1 2) (3)(4)(5 6)(7)

#### 1. [10 points] Sipser book, Exercise 2.2:

- 2.2 a. Use the languages  $A = \{a^m b^n c^n \mid m, n \geq 0\}$  and  $B = \{a^n b^n c^m \mid m, n \geq 0\}$  together with Example 2.36 to show that the class of context-free languages is not closed under intersection.
- b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.
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#### 2. [15 points] Sipser book, Exercise 2.13:

- 2.13 Let  $G = (V, \Sigma, R, S)$  be the following grammar.  $V = \{S, T, U\}$ ;  $\Sigma = \{0, \#\}$ ; and  $R$  is the set of rules:

$$\begin{aligned} S &\rightarrow TT \mid U \\ T &\rightarrow 0T \mid T0 \mid \# \\ U &\rightarrow 0U00 \mid \# \end{aligned}$$

- a. Describe  $L(G)$  in English.
- b. Prove that  $L(G)$  is not regular.
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#### 3. [30 points]

Give a context free grammar that generates the language  $\{x \in \{0, 1\}^* \mid \#_0(x) = \#_1(x)\}$ .

**Prove your answer.** (Show that your grammar generates every such string, and also that it generates only such strings. This may require a separate proof for *each direction* of set inclusion.)

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#### 4. [15 points] Sipser book, Problem 2.10, except give a **formal** description as well.

- 2.10 Give an informal description of a pushdown automaton that recognizes the language  $A$  in Exercise 2.9.

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

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#### 5. [10 points] Sipser book, Problem 2.14:

- 2.14 Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \varepsilon \\ B &\rightarrow 00 \mid \varepsilon \end{aligned}$$

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**6. [5 points] Sipser book, Problem 2.15:**

**2.15** Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let  $A$  be a CFL that is generated by the CFG  $G = (V, \Sigma, R, S)$ . Add the new rule  $S \rightarrow SS$  and call the resulting grammar  $G'$ . This grammar is supposed to generate  $A^*$ .

**7. [15 points] Sipser book, Problem 2.31:**

**2.31** Let  $B$  be the language of all palindromes over  $\{0,1\}$  containing an equal number of 0s and 1s. Show that  $B$  is not context free.