Computer Science 81, Spring 2010
Assignment 12
Due on or before Friday April 30

All proofs are informal and intended to develop intuition and expository style, so please provide convincing write-ups of each.

1. **[5 points]** What is the smallest alphabet Σ such that all Turing machines can be encoded as members of Σ^*?

2. **[20 points]** Consider the language of Turing machine descriptions
   $L = \{\langle M \rangle \mid M$ accepts at least 999 strings$\}$. Is L decidable, recognizable, or neither? Prove your answer.

3. **[10 points]** Suppose $R \subset \Sigma^*$ is a regular language other than $\varnothing$ and Σ^*. Let $L \subseteq \Sigma^*$ be any decidable language. Show that $L \leq_m R$ (L is mapping-reducible to R).

4. **[5 points]** Regarding the previous problem, is the same reduction true if L is recognizable but not decidable? Why or why not?

5. **[20 points]** Consider the language of Turing machine descriptions
   $L = \{\langle M \rangle \mid M$ accepts all strings of length 999 or longer$\}$. Is L decidable, recognizable, or neither? Prove your answer.

6. **[15 points]** Show that the special case of PCP (Post's Correspondence Problem) over a 1-letter alphabet is decidable.

7. **[10 points]** Show that a language L is recognizable iff $L \leq_m A_{TM}$ (the acceptance language for Turing machines).

8. **[15 points]** Consider the language of Turing machine descriptions
   $L = \{\langle M, w \rangle \mid M$ accepts w using at most 999 tape cells$\}$. Show that L is decidable. (Hint: For how many steps can M run without either accepting, rejecting, using more than 999 cells or going into an infinite loop?)