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1: m0>0∧n0>0
2: m0>0
3: n0>0
4: gcd(m0,n0)defined
5: gcd(m0,n0)=gcd(m0,n0)
6: gcd(m0,n0)=gcd(m0,n0)∧m0>0∧n0>0
7: m0>0∧n0>0→gcd(m0,n0)=gcd(m0,n0)∧m0>0∧n0>0
8: {gcd(m0,n0)=gcd(m0,n0)∧m0>0∧n0>0}
  (m:=m0){gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
9: {m0>0∧n0>0}(m:=m0){gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
10: {gcd(m,n0)=gcd(m0,n0)∧m>0∧n0>0}(n:=n0){gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
11: {m0>0∧n0>0}(m:=m0;n:=n0){gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
12: gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)
13: gcd(m,n)=gcd(m0,n0)
14: m>0
15: n>0
16: ¬(m=n)
17: m<n
18: gcd(m,n-m)=gcd(m0,n0)
19: n-m>0
20: gcd(m,n-m)=gcd(m0,n0)∧m>0∧n-m>0
21: m<n→gcd(m,n-m)=gcd(m0,n0)∧m>0∧n-m>0
22: ¬(m<n)
23: gcd(m-n,n)=gcd(m0,n0)
24: m>n
25: n<m
26: m-n>0
27: gcd(m-n,n)=gcd(m0,n0)∧m-n>0∧n>0
28: ¬(m<n)→gcd(m-n,n)=gcd(m0,n0)∧m-n>0∧n>0
29: ¬(m<n)→gcd(m,n-m)=gcd(m0,n0)∧m>0∧n-m>0
  ∧(¬(m<n)→gcd(m-n,n)=gcd(m0,n0)∧m-n>0∧n>0)
  gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)
30: ¬(m<n)→gcd(m,n-m)=gcd(m0,n0)∧m>0∧n-m>0
  ∧(¬(m<n)→gcd(m-n,n)=gcd(m0,n0)∧m-n>0∧n>0)
31: {gcd(m,n-m)=gcd(m0,n0)∧m>0∧n-m>0}
  (n:=n-m){gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
32: {gcd(m-n,n)=gcd(m0,n0)∧m-n>0∧n>0}
  (m:=m-n){gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
  {(m<n→gcd(m,n-m)=gcd(m0,n0)∧m>0∧n-m>0)∧(¬(m<n)
  →gcd(m-n,n)=gcd(m0,n0)∧m-n>0∧n>0)}if m<n then n
  :=n-m else m:=m-n fi{gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
34: {gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)}
  if m<n then n:=n-m else m:=m-n fi{gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
35: gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)
36: m>0
37: n>0
38: m+n>0
39: gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)→m+n>0
40: integer Km
41: gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)∧m+n=Km
42: m>0
43: n>0
44: ¬(m=n)
45: m+n=Km
46: m<n
47: m+(n-m)<Km
48: m<n-m+(n-m)<Km
49: ¬(m<n)
50: m>n
51: m-n+K<Km
52: ¬(m<n)→m-n+K<Km
53: (m<n-m+(n-m)<Km)∧(¬(m<n)→m-n+K<Km)
54: gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)∧m+n=Km
  ¬(m<n-m+(n-m)<Km)∧(¬(m<n)→m-n+K<Km)
55: {m+(n-m)<Km}(n:=n-m){m+n<Km}
56: {m-n+K<Km}(m:=m-n){m+n<Km}
57: {(m<n-m+(n-m)<Km)∧(¬(m<n)→m-n+K<Km)}
  if m<n then n:=n-m else m:=m-n fi{m+n<Km}
58: {gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)∧m+n=Km}
  if m<n then n:=n-m else m:=m-n fi{m+n<Km}
  {gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
59: while ¬(m=n)do if m<n then n:=n-m else m:=m-n fi od
  {gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)}
60: gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)
61: gcd(m,n)=gcd(m0,n0)
62: ¬(m=n)
63: m=n
64: gcd(m,n)=m
65: gcd(m0,n0)=m
66: m=gcd(m0,n0)
67: gcd(m,n)=gcd(m0,n0)∧m>0∧n>0∧¬(m=n)→m=gcd(m0,n0)
68: {gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}
  while ¬(m=n)do if m<n then n:=n-m else m:=m-n fi od{m=gcd(m0,n0)}
69: {m0>0∧n0>0}(m:=m0;n:=n0){gcd(m,n)=gcd(m0,n0)∧m>0∧n>0}while
  ¬(m=n)do if m<n then n:=n-m else m:=m-n fi od{m=gcd(m0,n0)}

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assumption
∧ elim 1
∧ elim 1
m0>0, n0>0 ⊢ gcd... 2,3
A=A 4
∧ intro 5,2,3
→ intro 1-6
variable-assignment
consequence(L) 7,8
variable-assignment
sequence 9,10
assumption
∧ elim 12
∧ elim 12
∧ elim 12
∧ elim 12
assumption
gcd(m,n)=X ⊢ gcd(m,...) 13
m<n ⊢ n-m>0 17
∧ intro 18,14,19
→ intro 17-20
assumption
gcd(m,n)=X ⊢ gcd(m,...) 13
¬(m=n), ¬(m<n) ... 16,22
A<B∧B>A 24
m<n ⊢ n-m>0 25
∧ intro 23,26,15
→ intro 22-27
∧ intro 21,28
→ intro 12-29
variable-assignment
variable-assignment
choice 31,32
consequence(L) 30,33
assumption
∧ elim 35
∧ elim 35
m>0, n>0 ⊢ m+n... 36,37
→ intro 35-38
assumption
∧ elim 41
∧ elim 41
∧ elim 41
∧ elim 41
assumption
m+n=Km, m>0 ⊢ ... 45,42
→ intro 46-47
assumption
¬(m=n), ¬(m<n) ... 44,49
m+n=Km, n>0 ⊢ ... 45,43
→ intro 49-51
∧ intro 48,52
→ intro 41-53
variable-assignment
variable-assignment
choice 55,56
consequence(L) 54,57
while 34,39,40-58
assumption
∧ elim 60
∧ elim 60
→E ⊢ E 62
m=n ⊢ gcd(m,n)=m 63
A=B, A=C ⊢ B=C 61,64
A=B∧B=A 65
→ intro 60-66
consequence(R) 59,67
Ntuple 11,68

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Provided:
DISTINCT gcd, m, m0, n, n0