Language Parsing

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Parsing

- Parsing is the process of determining whether a string is in a given language.
- Every compiler and interpreter does some form of parsing.
- Closely related is the process of assigning a meaning to the strings that are in the language.
- Refer to CS 60 materials for a review of how this can be done.

Parsing with a Pushdown Automaton

- We have seen how pushdown automata parse languages represented by context-free grammars.
- In some cases, the corresponding automaton is non-deterministic, which limits the practical use of pda's as a programming technique.
- Only a proper subset of languages can be parsed **deterministically** by a pda. Intuitively, these Deterministic Context-Free Languages are the ones that don't require "guessing".

Use of End-Markers

- In general, a pda will tell us when to accept the string parsed **so far**, as if there might be more coming.
- Sometimes we want to tell the pda that we **don't want an answer** to this question until all input has been supplied.
- One way to do this is to include an **endmarker**, say $, in the language being defined. The language will have the form
  {x$ | x has some property}
- The pda can react to the endmarker when it sees it.

Deterministic Parsing using Look-ahead

- A non-deterministic pda can sometimes be rendered "deterministic" if there is some way to restrict the choice of moves to at most one.
- The best-known cases are to use "look-ahead":
  - By using **knowledge of the next symbol in the input**, only certain rules (corresponding to certain grammar productions) can be seen to be applicable. The ability to do this depends on having the appropriate grammar. **The knowledge itself is outside the grammar**.
  - The LL(1) grammars can be parsed **top-down** using produce-match with one symbol look-ahead.
  - The LR(1) grammars can be parsed **bottom-up** using shift-reduce with one symbol look-ahead.
  - The concept of LL(k) and LR(k) are due to Donald Knuth.

LL(1) grammars

- For each production $A \rightarrow \gamma$ a set of terminal symbols
  Lookahead($A \rightarrow \gamma$)
  is **computed**. These are the symbols that could possibly occur **after A** in a sentential form.
- If no two productions with A as the LHS have symbols in common in their Lookahead sets, then the grammar is LL(1).
- In this case, knowledge of Lookahead($A \rightarrow \gamma$) tells us whether to apply production $A \rightarrow \gamma$ in top-down parsing.
- An LL(1) grammar is not necessarily the most natural. It may take a transformation to get an equivalent LL(1) grammar, or there may be none.

Example

- $E \to T \mid E + T$
- $T \to F \mid T * F$
- $F \to a \mid (E)$
- is **not** LL(1), since e.g. Lookahead($E \to T$) = {a, (} and Lookahead($E \to E + T$) = {a, (}
- So we'd need to transform this grammar.

Example

- $E \to T\,E'$
- $E' \to + T\,E' \mid \Lambda$
- $T \to F\,T'$
- $T' \to * F\,T' \mid \Lambda$
- $F \to a \mid (E)$
- **is** LL(1) , e.g.
  Lookahead($E' \to + T\,E'$) = {+} but
  Lookahead($E' \to \Lambda$) = {), \$}
  Lookahead($T' \to * F\,T'$) = {*} but
  Lookahead($T' \to \Lambda$) = {), +, \$}

LR(1)

- Handles a broader family of languages than LL(1) can.
- Bottom-up, shift-reduce parsing.
- Difficult to transform.
- Automated tools such as YACC can help.

CYK Algorithm

- CYK = "Cocke, Younger, Kasami", independent discoverers of the same algorithm.
- Deterministic, not using pda model.
- Does not require much transformation of the grammar.

CYK Motivation

- Problem is to determine whether a given string x is in L(G).
- A **naïve** approach, would be to generate all possible strings, in increasing length, until either:
  - x is generated, **or**
  - alll strings of length x or shorter have been generated.
- This is a very **slow** process; could be exponential time in the length of the string (e.g. if each symbol could have been generated by two different productions: 2x2x…x2 n times).

CYK Motivation

- Rather than working from the start symbol toward generated strings, might be better to work from string backward, to see if start symbol could have generated the string.
- This too could be exponential if not done carefully.
- The CYK uses "dynamic programming" to make the process efficient.

Dynamic Programming?

- Recursive expressions, such as
  $f(n) = f(n-1) + f(n-3)$; $f(n) = 1$ if $n < 3$;
  while **very clear** in their intent, may be **inefficient** if taken literally.
- They may entail much **recomputation** unless care is taken to ensure otherwise.
- The idea of dynamic programming is to compute from the "**bottom up**":
  f(0), f(1), f(2), f(3), …
  remembering values for **posterity** even if they might not be used ultimately.

CYK Algorithm

- Let $x = x_1 x_2 \ldots x_n$ be the string to be parsed.
- Define a(i, j) =

  $\{B \mid B \Rightarrow^* x_i x_{i+1} \ldots x_j\}$

  for each $i, j \in \{1, 2, \ldots, n\}$ with $i \leq j$.

- So $x \in L(G)$ iff $S \in a(1, n)$.
- The essence of CYK is how to compute a(1, n) efficiently.

CYK Algorithm

- Assume that the grammar is Chomsky Normal Form.
- We arrive at a(1, n) by the following method:
  - First compute a(i, i) for each i:

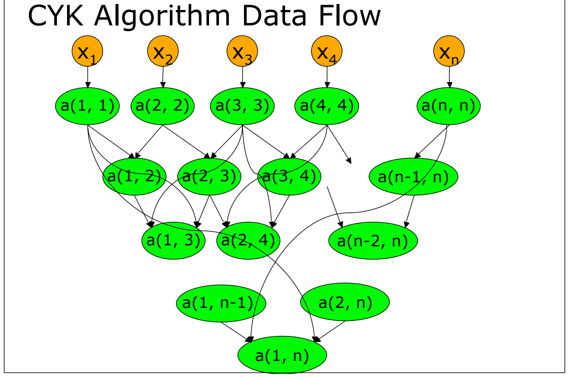
    $a(i, i) = \{B \mid B \rightarrow x_i\}$

- Since the grammar is in CNF, the indicated productions are the only ones that produce terminal symbols.

CYK Algorithm

- Next compute a(i, i+1) for each i, then a(i, i+2) for each i, and so on.
- To compute a(i, j) in general, we use pairs, which have been computed in the previous iteration:
  a(i, i) a(i+1, j)
  a(i, i+1) a(i+2, j)
  a(i, i+2) a(i+3, j)
  …
  a(i, j-1) a(j, j)
- For each pair, we need only to check whether there is a production of the form $A \rightarrow BC$ where $B \in a(i, i+k)$ and $C \in a(i+k-1, j)$ since the grammar is in CNF.

CYK Algorithm Data Flow



CYK "Wavefront" Matrix

a(1, 1)	a(1, 2)	a(1, 3)	a(1, 4)		a(1, n-1)	a(1, n)
	a(2, 2)	a(2, 3)	a(2, 4)		a(2, n-1)	a(2, n)
		a(3, 3)	a(3, 4)		a(3, n-1)	a(3, n)
			a(4, 4)			
					a(n-1, n-1)	a(n-1, n)
						a(n, n)

Each entry is computed from entries in its same row and column, e.g. a(1, 4) from a(1,1) and a(2, 4), a(1, 2) and a(3, 4), a(1, 3) and a(4, 4).

CYK Example

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

- This grammar is in CNF.

CYK Movie on input (()())

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input (()())

L					
	L				
		R			
			L		
				R	
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input (()())

L	∅				
	L	S			
		R	∅		
			L	S	
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input (()())

L	∅	∅			
	L	S	∅		
		R	∅	∅	
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input (()())

L	∅	∅	∅		
	L	S	∅	S	
		R	∅	∅	∅
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input (()())

L	∅	∅	∅	∅	
	L	S	∅	S	T
		R	∅	∅	∅
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input (()())

The string (()()) is accepted.

L	∅	∅	∅	∅	S
	L	S	∅	S	T
		R	∅	∅	∅
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input ())(())

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input ())(())

L	S				
	R	∅			
		R	∅		
			L	S	
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input ())(())

L	S	T			
	R	∅	∅		
		R	∅	∅	
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input ())(())

L	S	T	∅		
	R	∅	∅	∅	
		R	∅	∅	∅
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input ())(())

L	S	T	∅	∅	
	R	∅	∅	∅	∅
		R	∅	∅	∅
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input ())(())

The string ())(()) is not accepted.

L	S	T	∅	∅	∅
	R	∅	∅	∅	∅
		R	∅	∅	∅
			L	S	T
				R	∅
					R

- S → LT
- T → SR
- S → LR
- S → SS
- L → (
- R →)

CYK Movie on input abbaa (different grammar)

- S → AB
- S → BC
- A → AB
- A → a
- B → AA
- B → CB
- B → b
- C → a
- C → b

CYK Movie on input abbaa (different grammar)

- S → AB
- S → BC
- A → AB
- A → a
- B → AA
- B → CB
- B → b
- C → a
- C → b

A, C				
	B, C			
		B, C		
			A, C	
				A, C

CYK Movie on input abbaa (different grammar)

- S → AB
- S → BC
- A → AB
- A → a
- B → AA
- B → CB
- B → b
- C → a
- C → b

A, C	S, A, B			
	B, C	S, B		
		B, C	S	
			A, C	B
				A, C

CYK Movie on input abbaa (different grammar)

- S → AB
- S → BC
- A → AB
- A → a
- B → AA
- B → CB
- B → b
- C → a
- C → b

A, C	S, A, B	S, A, B		
	B, C	S, B	S	
		B, C	S	B
			A, C	B
				A, C

CYK Movie on input abbaa (different grammar)

- S → AB
- S → BC
- A → AB
- A → a
- B → AA
- B → CB
- B → b
- C → a
- C → b

A, C	S, A, B	S, A, B	S, B	
	B, C	S, B	S	B
		B, C	S	B
			A, C	B
				A, C

CYK Movie on input abbaa (different grammar)

The string abbaa is accepted.

- S → AB
- S → BC
- A → AB
- A → a
- B → AA
- B → CB
- B → b
- C → a
- C → b

A, C	S, A, B	S, A, B	S, B	S, A, B
	B, C	S, B	S	B
		B, C	S	B
			A, C	B
				A, C

Complexity of CYK

- n is the length of the input string.
- p is the number of productions, which can be treated as a constant.
- There are $O(n^2)$ sets to be computed.
- Each set can be represented as a **bit-vector**, so that elements can be added and membership-tested in O(1) time.
- A general set can be computed in O(n) iterations, each iteration involving examination of p productions.
- The total time is proportional to pn $n^2 \in O(n^3)$.

Expressing CYK Iteratively

⊗ is pairing

- Work by super-diagonals d = 2, 3, …, n
- d = 2 compute:
  - a(1, 2) = a(1, 1)⊗a(2, 2)
  - a(2, 3) = a(2, 2)⊗a(3, 3)
  - …
  - a(n-1, n) = a(n-1, n-1)⊗a(n, n)
- d = 3 compute:
  - a(1, 3) = a(1, 1)⊗a(2, 3) ∪ a(1, 2)⊗a(3, 3)
  - a(2, 4) = a(2, 2)⊗a(3, 4) ∪ a(2, 3)⊗a(4, 4)
  - …
  - a(n-2, n) = a(n-2, n-2)⊗a(n-1, n) ∪ a(n-2, n-1)⊗a(n, n)
- . . .
- d = n compute:
  - a(1, n) = a(1, 1)⊗a(2, n) ∪ a(1, 2)⊗a(3, n) ∪ . . .
    a(1, n-1)⊗a(n-1, n)

Summary of CYK

- Grammar is in CNF, input is $x = x_1 x_2 \ldots x_n$.
- **For** r = 1 to n // diagonal
  a(r, r) = {B | B → x_r}
- **For** d = 2 to n // super-diagonals
  For r = 1 to n-d+1 // row
  c = r+d-1; // column
  a(r, c) = ∅; // entry to compute
  For k = r to c-1
  For each production B → CD
  If C ∈ a(r, k)
  and D ∈ a(k+1, c)
  add B to a(r, c);
- x is in the language iff S ∈ a(n, n).

Verification by "Pedestrian Simulation"

```
main()
{
int n = 5;
int r, c, d, k;

for( d = 2; d <= n; d++ )
 {
 std::cout << "------------------------------" << std::endl;
 std::cout << "diagonal = " << d << std::endl;
 for( r = 1; r <= n-d+1; r++ )
  {
  c = r + d - 1;

  std::cout << std::endl;
  std::cout << "To compute a(" << r << ", " << c << ") use: " << std::endl;

  for( k = r; k < c; k++ )
   {
   std::cout << "a(" << r << ", " << k << ") paired with "
            << "a(" << (k+1 ) << ", " << c << ")" << std::endl;
   }
  }
 }
}
```

Verification by "Pedestrian Simulation"

----
diagonal = 3

To compute a(1, 3) use:
a(1, 1) paired with a(2, 3)
a(1, 2) paired with a(3, 3)

To compute a(2, 4) use:
a(2, 2) paired with a(3, 4)
a(2, 3) paired with a(4, 4)

To compute a(3, 5) use:
a(3, 3) paired with a(4, 5)
a(3, 4) paired with a(5, 5)

----
diagonal = 4

To compute a(1, 4) use:
a(1, 1) paired with a(2, 4)
a(1, 2) paired with a(3, 4)
a(1, 3) paired with a(4, 4)

To compute a(2, 5) use:
a(2, 2) paired with a(3, 5)
a(2, 3) paired with a(4, 5)
a(2, 4) paired with a(5, 5)

Additional Things to Ponder

- Simplify indexing expressions, e.g. by using transpose of matrix
- Recover derivation tree(s) from the algorithm

Real-World Parsing

- The value of CYK is an upper-bound for **arbitrary** context-free languages.
- $O(n^3)$ is too slow for programming languages, where $O(n)$ is desired (and doable for suitably-deterministic languages).
- May be acceptable for natural languages or other special languages.

Non-PL Applications of Parsing

- Natural Language
- Genomics (RNA strings)
  - Stochastic Context-Free Grammars
  - Transformational Grammars (transform one string to another)
- Art
  - Understanding a piece of art

Aside: L-Systems

- With grammars, productions are applied sequentially at any site in the sentential form where the LHS matches.
- With L-systems, productions are applied at all sites simultaneously, sort of like parallel processing or cellular automata.

Example: Fibonacci L-System

- Production
  - a → b
  - b → ba
- Derivation

a
b
ba
bab
babba
babbabab
babbababbabba
. . .

Example: Thue Sequence L-System

- Production
  - a → ab
  - b → ba
- Derivation

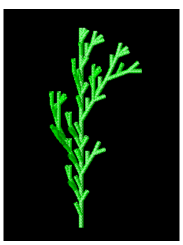
  ab
  abba
  abbabaab
  abbabaabbaababba
  abbabaabbaababbabaababbaabbabaab
  . . .

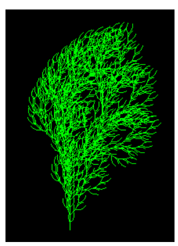
- Distinctions
  - Self-similar (fractal) nature.
  - There is a limit infinite sequence containing all strings as prefixes.
  - No string has a subsequence of the form xxx.

Applications of L-Systems?

- Studied extensively as models for biological development (particularly plants)
- Letters in L sequence can be interpreted as commands to add features, something like "turtle graphics"

Plants constructed using L-Systems





L-System Garden

More Examples:

- Przemyslaw Prusinkiewicz, The Algorithmic Beauty of Plants, Springer-Verlag, 1990.