# How Scheme (Racket) Thinks: Lists and Recursion

September 1, 2011

CS 42: Principles and Practice of Computer Science

Assignment #1 due Monday 11:59pm

`http://www.cs.hmc.edu/cs42`

`cs42help@cs.hmc.edu`

# Logging In: 1 name, 3 passwords

CIS Labs

Password?

CS Labs
(BK B102, BK B105)



Password?

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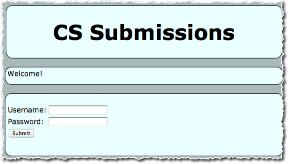
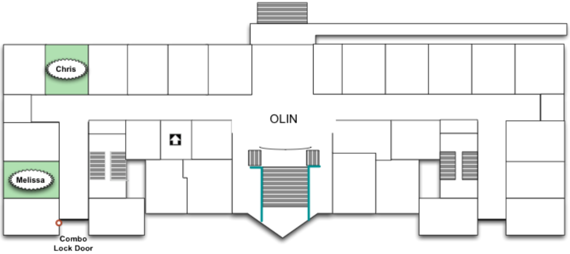
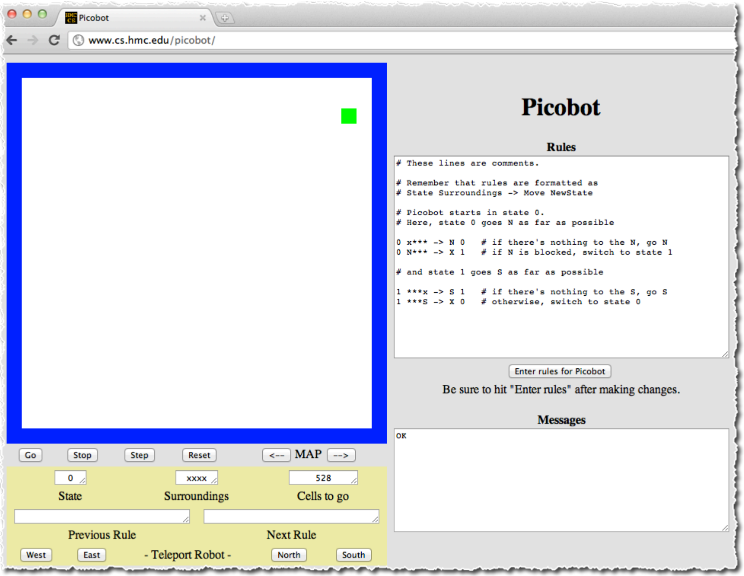



Password?

# Finding Me



# Recall: Picobot

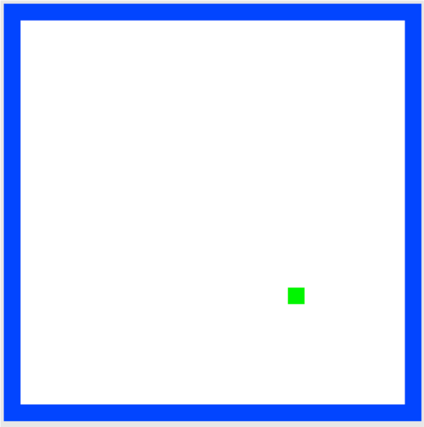


# "Quiz" 1 NAME:

Alter these "up and down" rules to make the robot traverse multiple columns, moving left

```
0 x*** -> N 0
0 N*** -> X 1


1 ***x -> S 1
1 ***S -> X 0
```

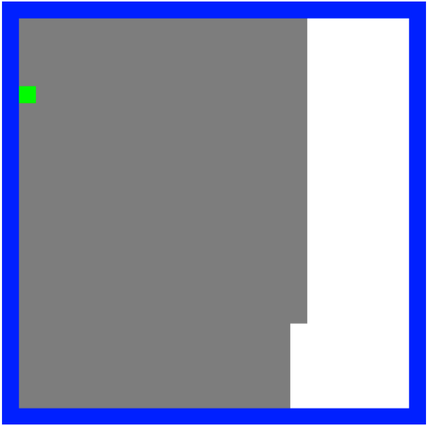


# "Quiz" 1 NAME:

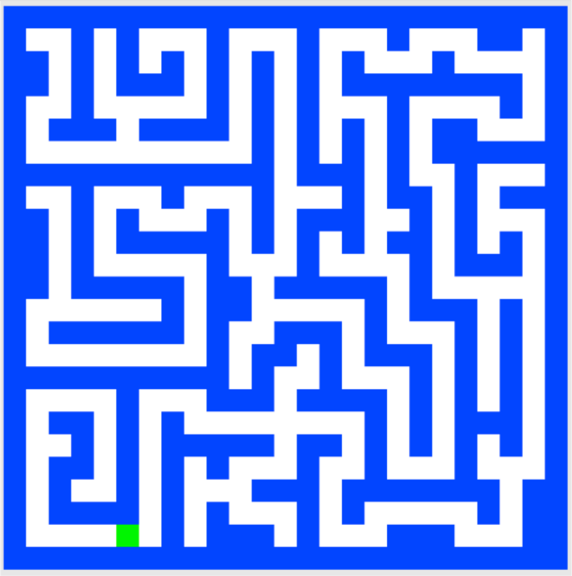
Alter these "up and down" rules to make the robot traverse multiple columns, moving left

```
0 x*** -> N 0
0 N*x* -> W 1
0 N*W* -> X 1
```

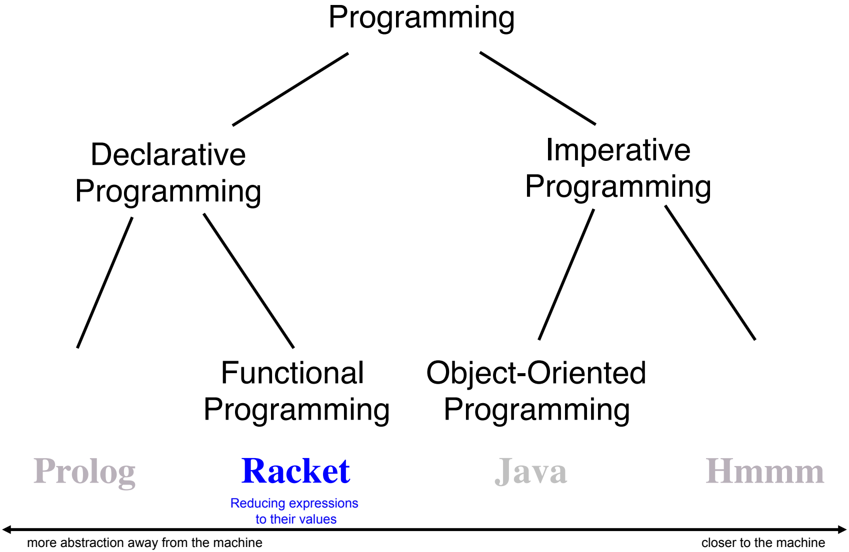
```
1 ***x -> S 1
1 **xS -> W 0
1 **WS -> X 0
```



# Strategies for Problem #2 ?



# Recall: Different Interpretations of "Computing"



# Racket: A *Functional* Language

## Functional Languages In General

- ✓ Computation model: *evaluating expressions* to get their result
  - Minimize "side effects."
  - Generally, $f(7) + f(7) = 2 * f(7)$
- ✓ High-Level Language: hides implementation details
  - Don't *get* direct access to memory
  - Don't *get* to decide how your data appears as bits and bytes
  - Don't *need* to worry about these!
  - Trades programmer time for run time. (But which is more expensive?)
- ✓ Functions as the primary building blocks
  - Toolkit approach: combine smaller functions into big ones
  - Functions are ordinary ("first class") values

# Racket: A Functional Language

## Racket in Particular

- ✓ Racket is a variant of Scheme
- ✓ Scheme is a "cleaned up" version of LISP, one of the very first programming languages!
- ✓ LISP was AI-inspired: lists are the only data structure, and almost everything is a list, including your program!
- ✓ Closely related to $\lambda$-Calculus, which predates computers!

# Questions to Ask When Learning a New Language

- ✓ What kinds of data are supported? (booleans, integers, ...)
- ✓ How do I give names to data?
- ✓ How do I call functions and built-in operations?
- ✓ How do I define my own functions?
- ✓ How do I make choices?

# What Kinds of Data?

| | |
|---|---|
| `#f    #t` | booleans |
| `42` | integers |
| `42.0` | floating-point (also: rational, complex) |
| `"this is a string"` | strings |
| `#\c` | characters |
| `'year   'x` | symbols |
| `'(a b c)  '(a "hi" (42 19.0))` | lists |

# How Do I Give Names To Data?

```
(define answer 42)

(define howdy "hello world!")
```

# How do I use built-in operations?

```
(+ 20 22)
(+ 60 (- 18))
(zero? (- 27 19))
```

Other Operators

| | |
|---|---|
| `and` `or` `not` | *boolean operators* |
| `+` `-` `*` `/` `modulo` `quotient` | arithmetic |
| `max` `min` `expt` `sqrt` `sin` `cos` ... | mathematical functions |

# How Do I Define My Own Functions?

```
(define (f N)
   (* N (+ N 1)))

(define (sum-upto N)
   (/ (f N) 2))
```

# Don't Quote Me On This!

```
'(f 10)
```

A list of length two, containing a symbol and an integer.

```
(f 10)
```

Code that runs function `f`, with the single input `10`.

# How Do You Make Choices?

```
(if b
  true-branch
  false-branch)
```

```
(cond
   [ test1    result1 ]
   [ test2    result2 ]
   [ else     result3 ]
)
```

Warning: what does this *expression* correspond to in Java/C/C++?

# Recursion

How can we take the power $\mathbf{b}^{\mathrm{N}}$?

```
; Given b and N, return b**N
(define (pow b N)
   (cond
      [                                     ] ; base case
      [else                                 ] ; recursive case
   ))
```

# Recursion

How can we take the power $\mathbf{b}^{\mathbf{N}}$?

```
; Given b and N, return b**N
(define (pow b N)
   (cond
      [                                          ] ; base case
      [else                                      ] ; recursive case
   ))
```

But I think we can do better!

# More Efficient Code?

```
; Given b and N, return b**N
(define (pow b N)
  (cond
     [ (= N 0)     1                      ]
     [ (odd? N)  (* b (pow b (- N 1))) ]
     [  else      (* (pow b (/ N 2))
                     (pow b (/ N 2)))   ]
     ))
```

# More Efficient Code?

```
; Given b and N, return b**N
(define (pow b N)
  (cond
     [ (= N 0)     1                        ]
     [ (odd? N)  (* b (pow b (- N 1))) ]
     [  else     (* (pow b (/ N 2))
                    (pow b (/ N 2)))   ]
     ))
```

But I think we can do better!

# `let` IS MORE!

`let*` allows you to name values temporarily:

```
(let* ( (y 3)
        (z 10) )
      (+ y z)
)
```

Local variables, but these variables don't vary!

# Final Definition

```
; Given b and N, return b**N
(define (pow b N)
  (cond
     [ (= N 0)    1                        ]
     [ (odd? N)  (* b (pow b (- N 1))) ]
     [  else     (let* ( (sqroot (pow b (/ N 2))) )
                        (* sqroot sqroot)) ]
     ))
```

# COMMENTS ON RECURSION?

*"Just pretend that the function you're writing already exists."*

*Geoff Romer (CS 60)*

*"To understand recursion, you must first understand recursion…"*

*Anonymous (CS 60)*

# Subtlety: `let*` vs. `let`

`let*` works sequentially, `let` works in parallel.

```
(let* ( (n 6)
        (m (+ n 1)) )
      (* n m))
```

```
(let ( (n 6)
       (m (+ n 1)) )
     (* n m))
```

```
(define x 6)
(define y 7)
(let* ( (x y)
        (y x) )
    (* x y))
```

```
(define x 6)
(define y 7)
(let ( (x y)
       (y x) )
    (* x y))
```

# Subtlety: `let*` vs. `let`

`let*` works sequentially.

`let` computes all the values, then creates the variables.

```
(let* ( (n 6)
        (m (+ n 1)) )
      (* n m))  ;; 42
```

```
(let ( (n 6)
       (m (+ n 1)) )
     (* n m))
```

```
(define x 6)
(define y 7)
(let* ( (x y)
        (y x) )
    (* x y))    ;; 49

;; now x is 6, y is 7
```

```
(define x 6)
(define y 7)
(let ( (x y)
       (y x) )
    (* x y))    ;; 42

;; now x is 6, y is 7
```

# Lists

```
(define M '(1 (2 3) 4))
```

Sequential View

```
(length M)

(first M)

(second M)

(third M)
```

Recursive View

```
(null? M)     ; (empty? M)

(first M)     ; (car M)

(rest M)      ; (cdr M)

(first (rest M))

(rest (rest M))           ;  '(4)
(rest (rest (rest M)))  ;  '()
```

The Fundamental List Dichotomy:
*every list is empty or non-empty.*

# Building Lists

```
(define L '(h a r))
(define M '(e v))
```

```
(list M 'u 'd 'd)

(cons 'k M)

(append L L)

(reverse M)

; How to get '(h a r v e y) ?
```

# Building Lists

```
(define L '(h a r))
(define M '(e v))
```

```
(list M 'u 'd 'd) ; ==> '((e v) u d d)

(cons 'k M)       ; ==> '(k e v)

(append L L)      ; ==> '(h a r h a r)

(reverse M)       ; ==> '(v e)

; How to get '(h a r v e y) ?
```

# Warning

```
(cons 5 '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)`
4. `'((5) (2 1))`
5. *error*

# Warning

```
(cons 5 '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)` ⟵
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(list '5 '(2 1))
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1. `'(5 (2 1))` ⟵
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# Warning

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(list '(5) '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)`
4. `'((5) (2 1))`
5. *error*

# Warning

```
(list '(5) '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)`
4. `'((5) (2 1))` ⟵
5. *error*

# Warning

```
(append 5 '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)`
4. `'((5) (2 1))`
5. *error*

# Warning

```
(append 5 '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)`
4. `'((5) (2 1))`
5. *error* ⟵

# Warning

```
(append '(5) '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)`
4. `'((5) (2 1))`
5. *error*

```
( reverse '((1 2) 3 4) )
```

# Warning

```
(append '(5) '(2 1))
```

---

1. `'(5 (2 1))`
2. `'((5) 2 1)`
3. `'(5 2 1)` ⟵
4. `'((5) (2 1))`
5. *error*

```
( reverse '((1 2) 3 4) )
```

# Recursion: Trials (and Tribulations)

```
; Given a list L, return its length.
; Already built-in, but let's define it anyway.
(define (length L)
   (cond
       [
       [



; This is a very common pattern!
```

# Recursion: Trials (and Tribulations)

```
; Given a list L, return its length.
; Already built-in, but let's define it anyway.
(define (length L)
   (cond
       [ (null? L)  0 ]
       [ else       (+ 1 (length (rest L))) ]
   ))

; This is a very common pattern!
```

# Recursion: Trials (and Tribulations)

```
; Given a value e and a list L, check if e is in L
; Already built-in, but let's define it anyway.
(define (member e L)
   (cond
       [
       [
```

```
;; the built-in member returns a list, rather than #t

;; Note: for "if" and "cond", anything that is not #f
;; is considered true, including lists (and the empty list!)
```

# Recursion: Trials (and Tribulations)

```
; Given a value e and a list L, check if e is in L
; Already built-in, but let's define it anyway.
(define (member e L)
   (cond
       [ (null? L)              #f ]
       [ (equal? e (first L))   #t ]
       [ else                   (member e (rest L)) ]))

;; the built-in member returns a list, rather than #t

;; Note: for "if" and "cond", anything that is not #f
;; is considered true, including lists (and the empty list!)
```

# "Quiz" 2

```
; Given L, return same list but with
; top-level elements in opposite order
(define (reverse L)
  (cond
```

```
; What do I compute?
(define (mystery N)
  (cond
     [ (< N 2) 0 ]
     [ else (+ 1
                (mystery (quotient N 2)))]
  ))
```

```
; Given L, flatten out all nesting
; e.g., (flatten '(1 (2 3 (4)) (5 6)))
;            ==>  '(1 2 3 4 5 6)
(define (flatten L)
  (cond
```

```
; given N>1, compute the smallest prime
; factor of N, e.g., (spf 45) ==> 3
(define (spf N)
```

# "Quiz" 2 Solutions

```
; Given L, return same list but with
; top-level elements in opposite order
(define (reverse L)
  (cond
     [ (null? L)  L ] ;; or return '()
     [ else (append (reverse (rest L))
                    (list (first L))) ]
  ))
```

Notes:

- ✓ *We can't use* **cons** *to put a single item at the* ***end*** *of a list.*
- ✓ *Append expects two lists, not a list and a single element. (That's why we have to build a list of length one, containing only* `first L`*):*

```
(reverse '(1 2 3))  ===>  append (reverse '(2 3)) '(1)
                    ===>  append '(3 2) '(1)
                    ===>  '(3 2 1)
```

# "Quiz" 2 Solutions

```
; Given L, flatten out all nesting
; e.g., (flatten '(1 (2 3 (4)) (5 6)))
;           ==>  '(1 2 3 4 5 6)
(define (flatten L)
  (cond
    [ (null? L)       L ]
    [ (not (list? L)) (list L) ]
    [ else  (append (flatten (first L)) (flatten (rest L))) ]
  )
)
```

# "Quiz" 2 Solutions

```
; What do I compute?
(define (mystery N)
  (cond
     [ (< N 2) 0 ]
     [ else (+ 1
               (mystery (quotient N 2)))]
  ))
```

The number of times N can be divided by 2, throwing away remainders, until we get below 2. That is, (an integer approximation to) the logarithm base 2 of N.

# Putting assoc in it

An association list (a-list) is a list of pairs.
It's a simple way to associate keys with values
(like dictionaries, lookup tables, ...).

```
(define rome '( (#\I 1) (#\V 5)  (#\X 10) (#\L 50)
                (#\C 100) (#\D 500) (#\M 1000) ))
```

```
(assoc #\D rome)   ;  ==>  '(#\D 500)
(assoc #\Z rome)   ;  ==>  #f
```

```
;; Note: for "if" and "cond", anything that is not #f
;; is considered true, including lists (and the empty list!)
```