Paths / Interpreters / Grammars

September 15, 2011

CS 42: Principles and Practice of Computer Science

Graph Reachability: Getting from a to b

One approach (among many!)

- ✓ Keep a collection of "partial" paths starting at a
  - For efficiency, we might represent a path as a list in *reverse* order.
- ✓ Repeatedly take out a path (the "active path")
  - If it ends at the destination b, we're done!
  - If not, find reasonable ways to make it one step longer, add them to our collection.

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What's "reasonable?"

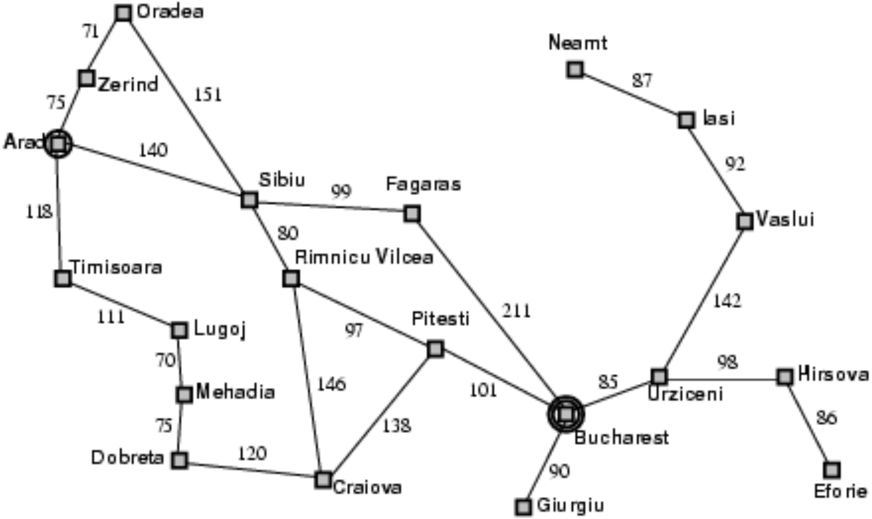
- ✓ Don't create paths with cycles (loops)
- ✓ Or, don't create new paths to places you've already reached. (Requires keeping track separately of which nodes have been reached.)

```
(define (reach a b G)
  (reach-help b (list (list a)) G))

(define (reach-help dst path-list G)
  (if (null? path-list)
      #f
      (let ((active-path (first path-list))
            (remaining-paths (rest path-list)))
        ; If active path contains the destination, return #t
        ; Otherwise, expand the active path by one node for
        ;  each kid, ignoring kids that create cycles, making
        ;  a list of 0 or more new paths.
        ; Append this new list of paths onto the remaining
        ;   paths and recurse.
        ; At least one helper function (to build the list
        ;   of new paths) is recommended.
        ...
```

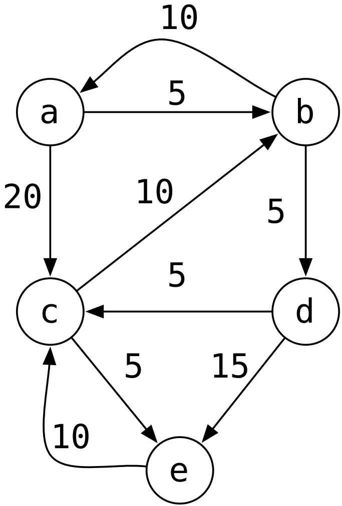
Weighted Graphs

Find the shortest path from Bucharest to Arad.



Dijkstra's Algorithm

Greedy/Best-First Search



One implementation strategy:

- ✓ A collection of partial paths (with lengths)
- ✓ Always pick the shortest partial path as your "active path".
- ✓ Stop when the *active path* reaches your destination.

Note:

- ✓ The first time an active path ends at any node n, this will be the shortest path from a to n.
- ✓ We could tweak the code to find shortest paths from a to all the nodes in the graph!

How to get from a to e?

The `minimath` language

An example interaction

```
Welcome to DrRacket, version 5.1.3 [3m].
Language: racket; memory limit: 128 MB.
> (read-eval-print)

input> 5 + 4
result: 9

input> 5 * 5
result: 25

input> 5 + 3 * 2
result: 11

input> (5 + 3) * 2
result: 16
```

Looks simple. How to implement it? What are the pieces?

Structure of a Generic Interpreter

1. Lexical Analyzer (a.k.a. Tokenizer, Lexer, Scanner)
   - Turns input (a linear sequence of characters) into a linear sequence of tokens/lexemes.
2. Parser
   - Turns the linear sequence of tokens into a structured representation of the program (usually a tree).
3. Evaluator
   - Process/interpret/run the structured representation to compute a final answer
   - May need to maintain an "environment" that associates program variables to their values.
4. Printer
   - Display the final result.

MINIMATH: 32+2 * 5

1. Lexical Analyzer (a.k.a. Tokenizer, Lexer, Scanner)

2. Parser

3. Evaluator

4. Printer

Racket: `(average 44 (+ 15 15))`

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Syntax vs. Semantics

Syntax vs. Semantics

- ✓ Syntax: Formation Rules
  - *Colorless green ideas sleep furiously.*
  - `(+ 3 true)`
- ✓ Semantics: Meaning

Specifying Syntax via CFGs

A *context-free grammar* is a set of rules for producing a set of strings (a *language*).

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$$
\begin{aligned}
S &\rightarrow V + S \mid V \\
V &\rightarrow 0 \mid 1 \mid 2 \mid \cdots \mid 9
\end{aligned}
$$

Ingredients:

- ✓ Nonterminals: S, V
- ✓ Terminals: $+$, 0, 1, 2, …, 9
- ✓ Production rules: (*see above*)
- ✓ Where to start: S

Show how to produce 4 starting from S.
Show how to produce 4 + 5 starting from S.
What other strings can we produce?

Using Structure to Clarify Meaning

He gave her cat food.

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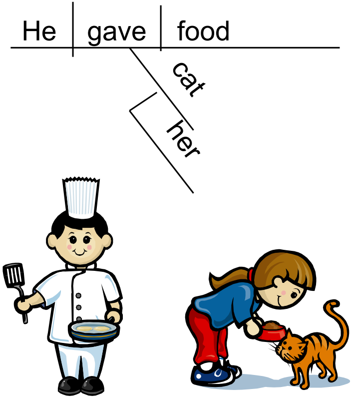

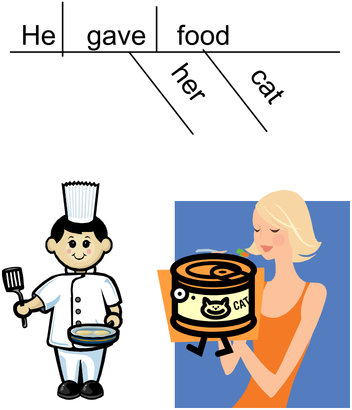

Parse Trees

The *parse tree* of a string makes explicit how a string was produced:

- ✓ Root is the start symbol
- ✓ When we apply a rule, items on the right-hand-side become children

Parse trees for 4 and 4+5?

$$
\begin{array}{lcl}
S & \rightarrow & V + S \mid V \\
V & \rightarrow & 0 \mid 1 \mid 2 \mid \cdots \mid 9
\end{array}
$$

Parse Trees

Show the parse tree for `9 - 3 + 2`

$$
\begin{aligned}
S &\rightarrow V + S \mid V - S \mid V \\
V &\rightarrow 0 \mid 1 \mid 2 \mid \cdots \mid 9
\end{aligned}
$$

Name:

"Quiz:" Grammars

$$
\begin{aligned}
P &\rightarrow S * P \mid S \,/\, P \mid V \\
S &\rightarrow V + S \mid V - S \mid V \\
V &\rightarrow 0 \mid 1 \mid 2 \mid \cdots \mid 9
\end{aligned}
$$

1. Draw the parse tree for

   `7 * 3 + 9 / 2`

2. How could we change the grammar to get a parse tree with the *usual* mathematical meaning of this expression?
3. What could we add to permit parentheses (and) to group sub-expressions?