Computers: What can't they do!
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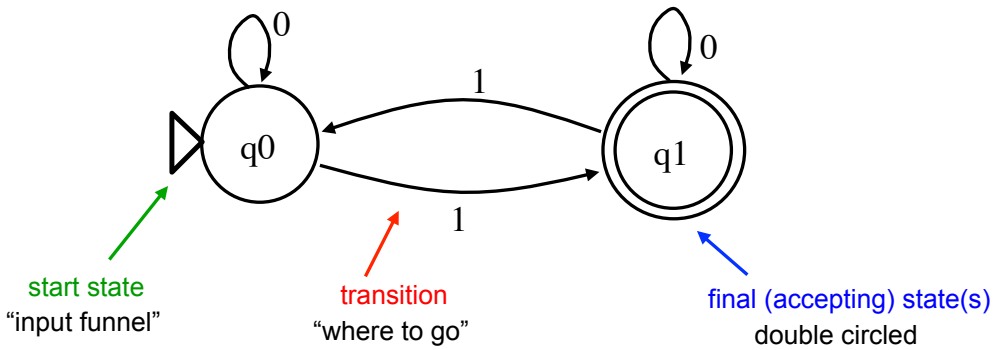
Part 2: NFAs and Regexps; Turing Machines

November 22, 2011

CS 42: Principles and Practice of Computer Science

The Story So Far...

- ✓ If we want to know what can/cannot be computed, it's enough to consider *decision problems*.
- ✓ Every decision problem corresponds to a *language*, a set of inputs (finite strings) where the answer is yes.
- ✓ DFAs (a.k.a. deterministic finite automata a.k.a. deterministic finite state machines) can model any deterministic mechanism with a finite number of configurations (finite memory).



A Jewel of Theoretical Computer Science



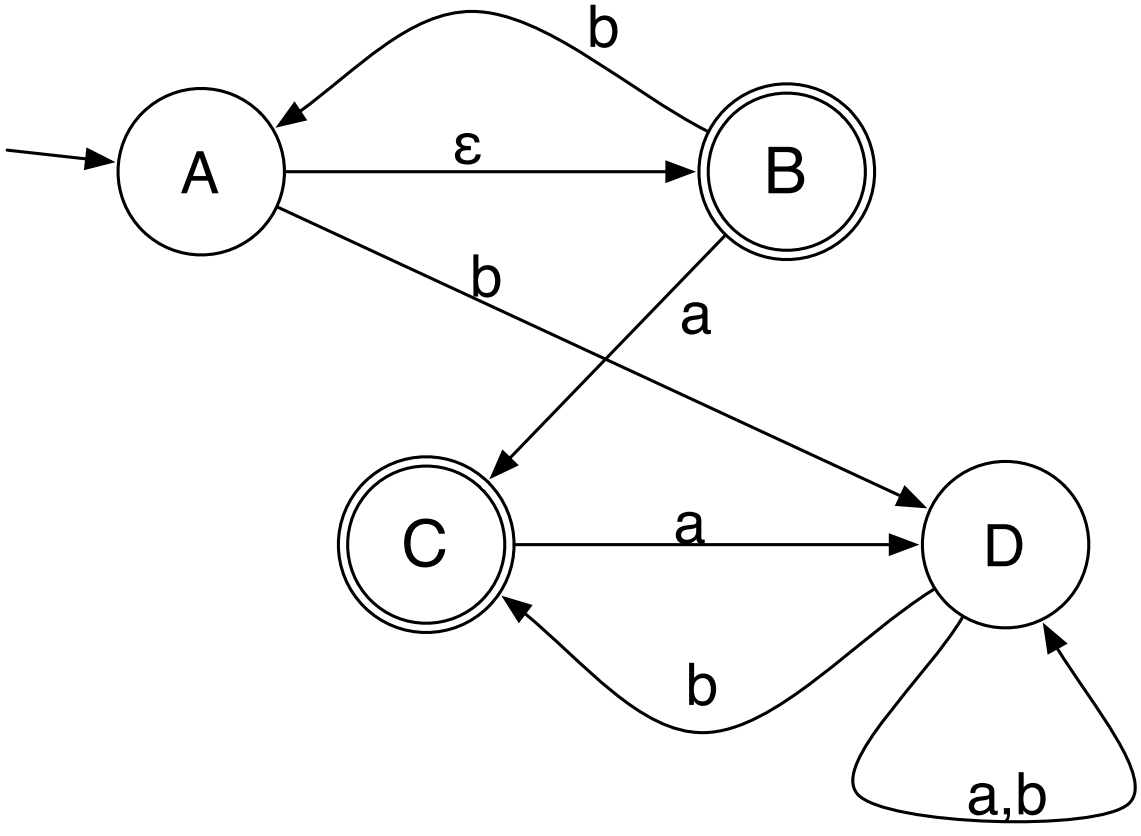
The following are equivalent:

- ✓ There is a DFA accepting the language L.
- ✓ [Rabin and Scott] There's an NFA accepting L.
- ✓ [Kleene] L can be described by a regular expression.

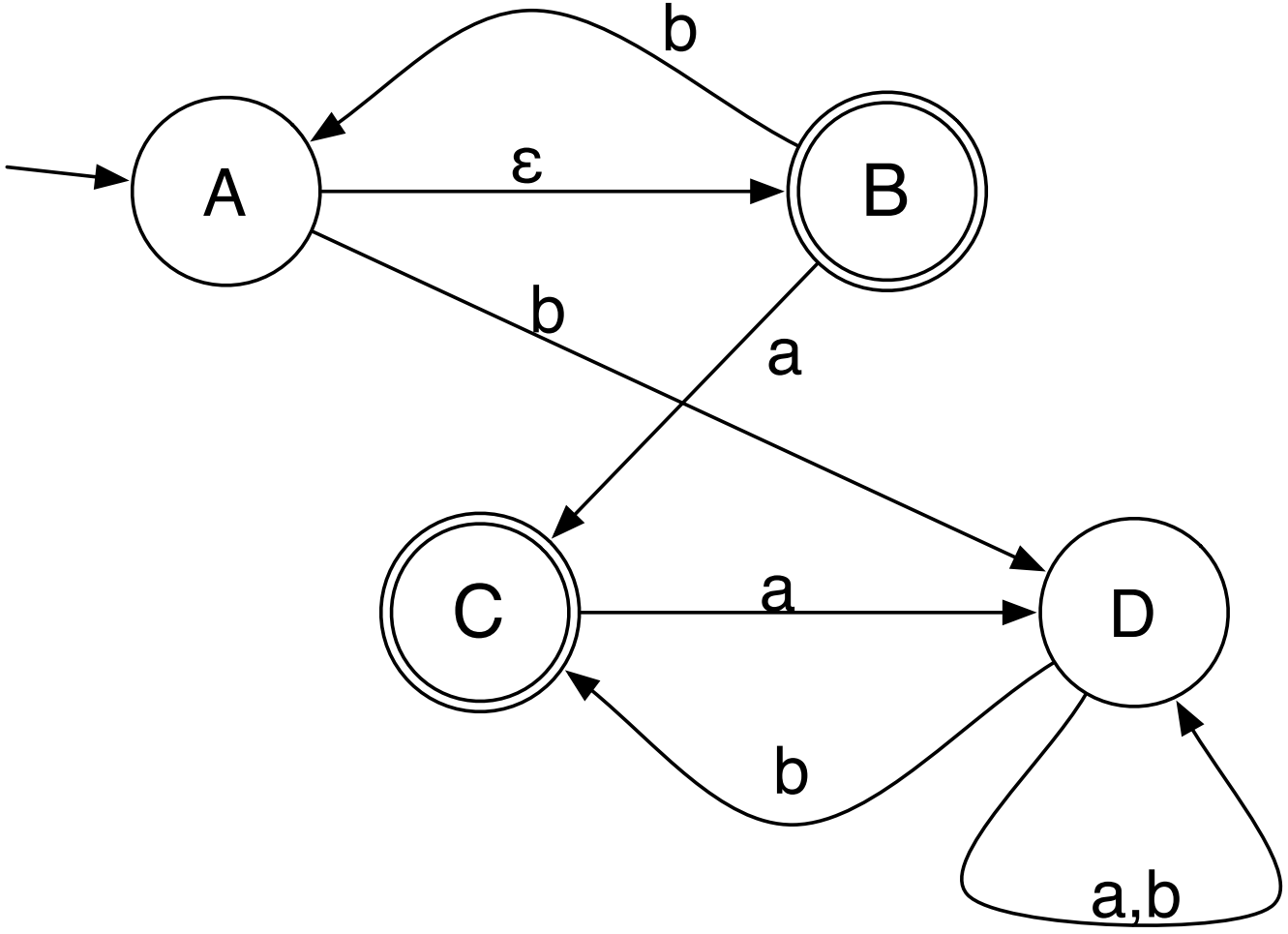
DFAs + Nondeterminism = NFAs

An NFA is a has the magic power to "guess" the right way to go, and can even change states without reading input (ϵ transitions).

NFA accepts the string if there exists at least one accepting path.



What's Accepted? (bb? b? aaab? ba?)



Regular Expressions: Simple Language Specifications

By *example*:

- ✓ $1^* \mid 1(00)^*$

$$\{ w \in \{0,1\}^* \mid w \text{ is a sequence of (zero or more) ones} \}$$
$$\cup$$
$$\{ w \in \{0,1\}^* \mid w \text{ is a } 1 \text{ followed by an even number of zeros.} \}$$

- ✓ $\mathtt{b}(\mathtt{ea}|\mathtt{a})(\mathtt{r}|\mathtt{d})$

$$\{\mathtt{bear}, \mathtt{bead}, \mathtt{bar}, \mathtt{bad}\}$$

- ✓ $(0|1)(0|1)1(0|1)^*$

$$\{ w \in \{0,1\}^* \mid w\text{'s third character is a } 1 \}$$

Regular Expression Ingredients

A regular expression s can be:

- ✓ $\emptyset$
- ✓ ϵ (or λ)
- ✓ A single character from the alphabet (e.g., a or 0)
- ✓ Concatenation: $r_1 r_2$
- ✓ Union: $r_1 \cup r_2$
- ✓ Repetition (Kleene Star): r_1^*.

Regular Expressions in Practice

Unix's **egrep** *command does* line-by-line *search for* text *matching a* regular *expression.*

```
egrep   'hh'        /usr/share/dict/words
egrep   'y.*y'      /usr/share/dict/words
egrep   '(xq|hq)'   /usr/share/dict/words
egrep   '^y.*y$'    /usr/share/dict/words
```

Many *systems, including* **egrep** *add extra operations* (mostly, but not entirely, *abbreviations*)

REs to the Rescue!

← PERL

practical extraction and report language

Email Address

```
\b[A-Z0-9._%-]+@[A-Z0-9.-]+\.[A-Z]{2,4}\b
```

Options: case insensitive

www.regular-expressions.info/regexbuddy/email.html

Google /* Code Search */ LABS

Google Code Search

About Google Code Search

Frequently Asked Questions

1. What kind of code are you crawling?
2. What regexp syntax does Code Search support?
3. What programming languages do you support?

but how does regular expression matching actually ***work***? →

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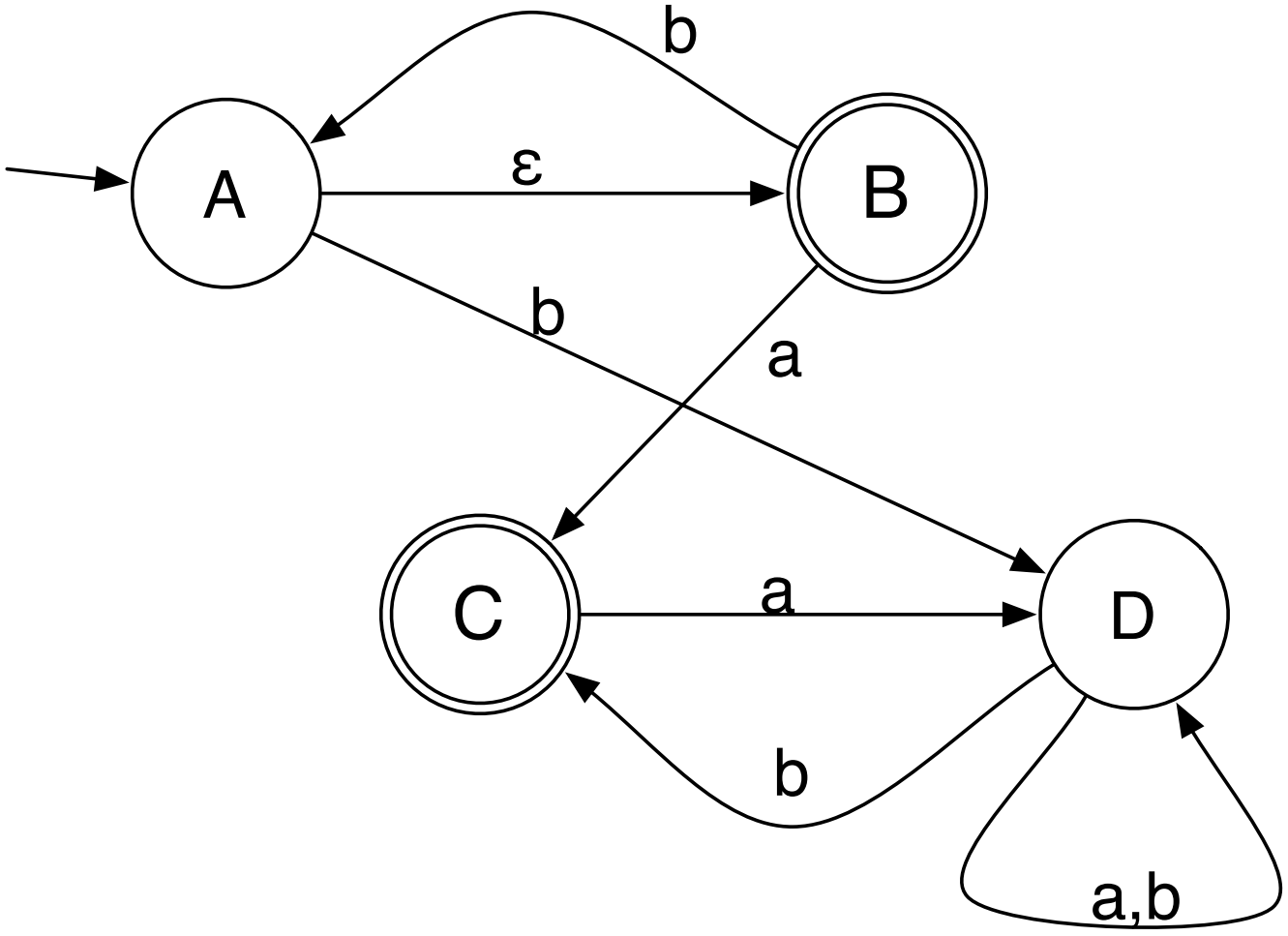
Can this be Simulated by a DFA?



Can we turn this Regular Expression into an NFA?

$$1^* \mid 1(00)^*$$

Not *covered* in this class: DFAs to *regular* expressions

Nonregular Language Theorem

Given a language L, two strings x and y are *distinguishable* if there is some string z such that $xz \in L$ and $yz \notin L$, or vice-versa.

If strings are distinguishable, running them through a state machine for L must lead you to *different* states.

If there is an infinite set of pairwise-distinguishable strings, then none of them can lead you to the same state-machine state. Thus, there cannot by a finite state machine recognizing L.

A language is *regular* if it can be recognized by a finite state machine, and *nonregular* otherwise.

Conclusions

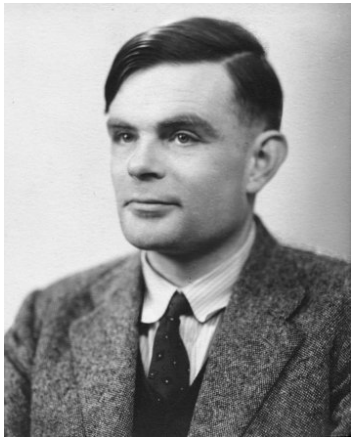
Any computer is a finite state machine.

Hence, there are questions it provably cannot answer. (E.g., "is the input a palindrome?")

But it sure seems like computers can answer these questions (until we run out of memory), so maybe this isn't so helpful in practice?

Perhaps we should consider more powerful models of computation.
(But not too powerful! Why?)

Alan Turing



WWII

Enigma machine ~ The axis's encryption engine

1946

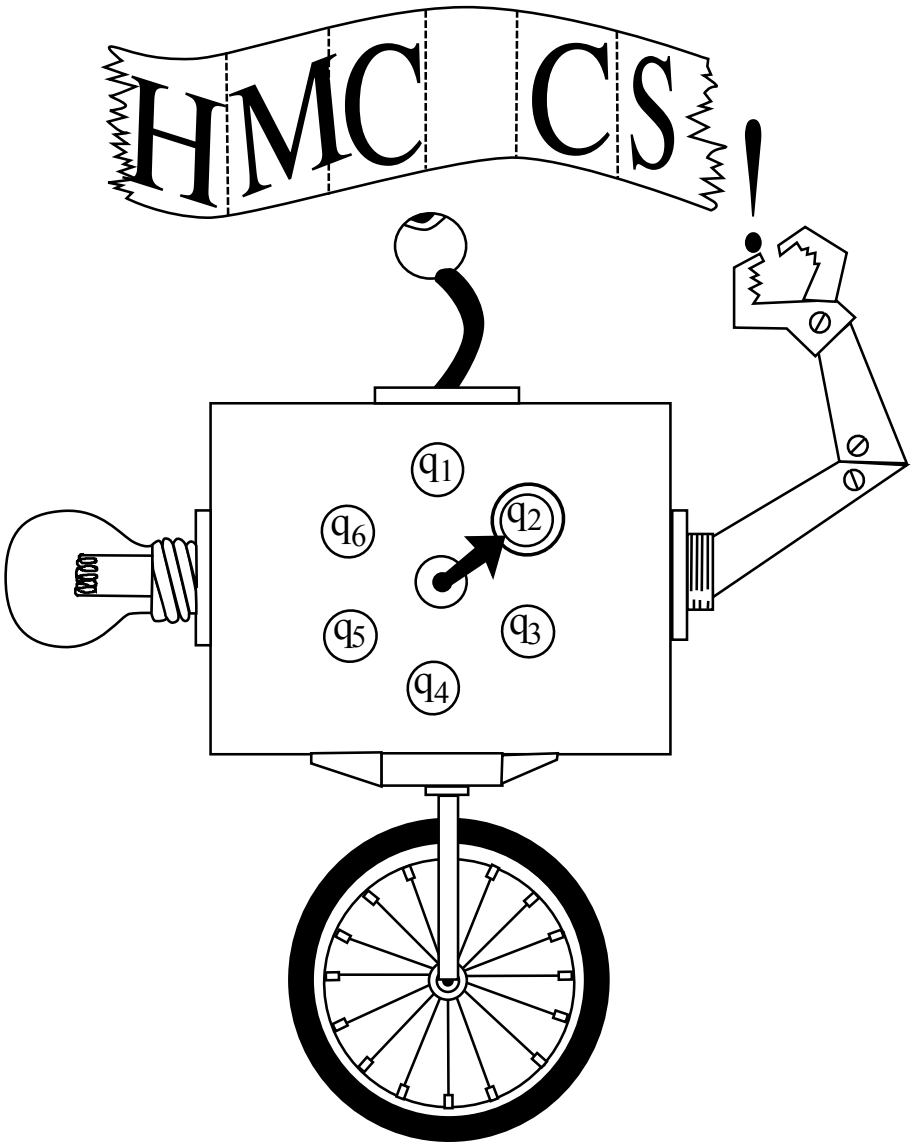
2007
Bletchley Park

Turing Machines

A simple model of universal computation.

- ✓ Control/Program: a finite state machine
- ✓ Data: An *infinite* tape, with a read/write head.
  Initially blank except for the input string.
- ✓ Steps determined by current state + current symbol;
  Can overwrite current symbol, and move left or right.
- ✓ Can stop (and say yes/no) or not.

Turing Machine: Artist's Conception



TM Demo(s)

http://ironphoenix.org/tril/tm/

http://www.youtube.com/watch?v=cYw2ewoO6c4