Assignment 10: Regular, Non-Regular, and Context-Free Languages

Due: Wednesday, November 16

- Emails about this assignment should be directed to `cs81help@cs.hmc.edu`.
- The usual collaboration rules apply. You may *discuss* an exercise with any other student(s) currently taking CS 81 as long as:
  - You contribute equally;
  - You come away from this discussion only with *understanding in your head* — no written materials or computer notes may be retained;
  - Your submission is authored solely by you, on a separate occasion.
- You should refer only to materials from this semester of CS 81 (lecture notes, handouts, textbooks, grutors, profs, etc.).
- Make sure your submission includes your name!

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1. Review Chapters 8 and 11.1–11.3 of Rich, paying particular attention to all the examples (in the shaded boxes). Also, read Appendix I.1 (up to I.1.4), and Appendix J.1, and Appendix O.

   Come up with one questions related to the reading. These may refer to points where the book is confusing, or simply to some question or conjecture that occurs to you while doing the reading.

   Come up with (at least) one question about the reading; these should be a question where you're not sure of the answer. It may relate to points where the book is confusing, or simply to some related question or conjecture that occurs to you while doing the reading.

2. Do Exercise 1(e,j,n,p,q,s) on pages 182–183 of Rich.

3. Do Exercise 7(a,c only) on page 183 of Rich. [Hint: you may assume you are given a DFA, NFA, or regular expression, whichever is most convenient; then show how to transform it.]

4. Let

$$\Sigma_3 := \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains eight size-3 columns of 0s and 1s. A string in symbols in Σ_3 thus builds three rows of 0s and 1s. Consider each row to be a binary number, and let

$$B := \{ w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows} \}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: show that *reverse*(B) is regular; by a previous problem, then so is *reverse*$($*reverse*$(B)) = B$.)

5. Let

$$\Sigma_2 := \{ \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right], \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right] \}.$$

Consider each of the two rows in Σ_2^* to be a binary number, and let

$$C := \{ w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is three times the top row} \}.$$

For example, $\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right] \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right] \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right] \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right] \in C$ but $\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right] \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right] \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right] \notin C$. Show that C is regular. (Hint: show that *reverse*(C) is regular.)

6. Let Σ_2 be as in the previous problem. Show that

$$D := \{ w \in \Sigma_2^* \mid \text{the top row of w is a strictly larger number than the bottom row} \}.$$

For example, $\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right] \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right] \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right] \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right] \in D$, but $\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right] \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right] \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right] \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right] \notin D$. Show that D is regular.

7. Prove that the following languages are context free, by giving a context-free grammar for each.

   (a) $\{ a^k a^j b^j c^k \mid j, k \geqslant 0 \}$

   (b) $\{ a^i b^j \mid i \neq j \ \wedge \ i, j > 0 \}$

8. Prove that the following languages are *not* context free:

   (a) $\{ a^n b^{2n} c^n \mid n \geqslant 0 \}$

   (b) $\{ a^{\max\{m,n\}} b^m c^n \mid n, m \geqslant 0 \}$