Assignment 7: Temporal Logic and Cardinality

Due: 11:59pm, Thursday, October 27, 2011

There is no class on Wednesday, October 26.
Put your solutions under the professor's door (Olin 1251) by Thursday.

- Emails about this assignment should be directed to `cs81help@cs.hmc.edu`.
- Grutors will be available in the Platt Living Room Sunday, Monday, and Tuesday from 8pm to 12.

1 Temporal Logic

(CTL was described in class. It also appears in Section 3.4 of Huth & Ryan.)

	Eventually True	Invariably True
Some Path	EF p Possibly	EG p Potentially Always
All Paths	AF p Inevitably	AG p Invariably

Consider the possible execution paths in the following piece of code:

```
int a[100] = { 0, 0, 0, ..., 0 };  // Array is initially all zeros

while (true) {
    int k = user_input();   // Assume this user input will be a number
                            //      between 0 and 99 inclusive!

    if (a[k] == 1) {
         for (int i = 0; i < 100; ++i)
              a[i] = 0;
    } else {
         a[k] = 1;
    }
}
```

Which of the following logical propositions are true (starting at the beginning of the program)? Explain your answer; you do not need to provide a full formal proof, but your explanation should be clear and convincing.

1. $\mathcal{A}\mathsf{G}\,(\mathsf{a}[42] = 0)$
2. $\mathcal{A}\mathsf{G}\left(\sum_{j=0}^{j<100} \mathsf{a}[j] < 100\right)$
3. $\mathsf{EF}\,(\mathsf{a}[42] = 1)$
4. $\mathcal{A}\mathsf{F}\,(\mathsf{a}[42] = 1)$
5. $\mathsf{EG}\,(\mathsf{a}[42] = 0)$
6. $\mathcal{A}\mathsf{G}\,(\mathcal{A}\mathsf{F}\,(\mathsf{a}[42] = 0))$
7. $\mathcal{A}\mathsf{G}\,(\mathsf{EF}\,(\mathsf{a}[42] = 1))$
8. $\mathsf{EF}\,(\mathcal{A}\mathsf{G}\,(\mathsf{a}[42] = 0))$

2 Cardinality

Carefully argue whether the following sets are countable or not.

1. The set of partial functions from $\mathbb{N}$ to $\mathbb{N}$ whose support is a finite set.

   [A total function is one that, for any input, produces an output. A *partial* function is one that, for some or all particular inputs, always returns "don't know" or "undefined" (often written $\perp$ although it's not a truth value) rather than returning an output. The *support* of a partial function (sometimes called the domain or domain of definition) is the collection of inputs that provide non-$\perp$ output.

   For example, we could have a partial function $\mathsf{sqrt} : \mathbb{R} \rightharpoonup \mathbb{R}$ defined by

   $$\mathsf{sqrt}(x) := \begin{cases} \sqrt{x} & \text{if } x \geqslant 0; \\ \perp & \text{otherwise.} \end{cases}$$

   The support of sqrt is then the set of all nonnegative real numbers.]

2. The set of (unlabeled) binary trees, where every node has either 0 or 2 children. (This was one of the sets defined inductively during the very first lecture of CS 81).

3. The set of flow networks with rational capacities (finite directed graphs, where each directed edge is labeled with a rational number called the "capacity" of that edge).

4. The set of Java programs that typecheck.