

CS 81

Computability and Logic

August 31, 2011

Please fill out the survey
and then skim the syllabus
while I'm taking pictures...

CS 81

Computability and Logic

An Introduction to Formal Systems

Proofs vs. Computation

Defining the System

Well-Formed Formulas,
Axioms/Inference Rules

Well-Formed Programs,
Language Definition

Specific Instances

Assumptions
Proof Step 1
Proof Step 2
...
Conclusion

Initial Conditions
Computation Step 1
Computation Step 2
...
Result

Logic: A Selective History

Aristotle

Boole

Frege

Russell

Hilbert

Gödel

Church/Turing

Logic?

This dog is a father

This dog is mine

Therefore, this dog is my father

Logic?

Prof. Ran is Prof. Libeskind-Hadas
This is Prof. Libeskind-Hadas

Therefore, this is Prof. Ran

Man is animal
Fluffy is animal

Therefore, Fluffy is Man

Logic?

The Steelworkers were unionized

Therefore, they had no missing electrons

Language is Ambiguous

John saw a picture
of the prettiest girl he had ever seen
hanging on a locker door.

Logic: A Selective History

Aristotle

Boole

Frege

Russell

Hilbert

Gödel

Church/Turing

Logic is Subtle

Put $U := \{ S \mid S \text{ is a set} \}$

Logic: A Selective History

Aristotle

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Frege

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Hilbert

Gödel

Church/Turing

An Inductively Defined Set

I'm thinking of a set S .

0 is in S .

If n is in S , then n' is in S .

S is the smallest set obeying these rules.

(Equivalently: every element of S can be shown to belong to S by using the above rules finitely many times.)

An Inductively Defined Set

$\overline{\underline{\underline{\quad}}}$ is in L .

If (n is an integer) and (l is in L),
then \longrightarrow

n	l
-----	-----

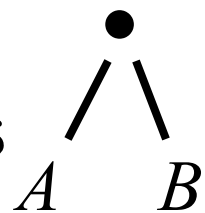
 is in L .

L is the smallest set obeying these rules.

(Equivalently: every element of L can be shown to belong to L by using the above rules finitely many times.)

An Inductively Defined Set

- is in T .

If A and B are in T , then so is 

T is the smallest set obeying these rules.

(Equivalently: every element of T can be shown to belong to T by using these rules finitely many times.)

Structural Induction

If the construction of an inductively defined set
guarantees/preserves some property

then

every element of the set has that property.

An Inductively Defined Set

$\exists l$ in L is finite? ✓

If n an integer and l in L is finite

then \rightarrow

n	l
-----	-----

 is finite? ✓

QED

Example: 2D Points

An inductive definition for a set S :

1. $(3, 5) \in S$.
2. If $(x, y) \in S$ then $(x + 2, y) \in S$.
3. If $(x, y) \in S$ then $(-x, y) \in S$.
4. If $(x, y) \in S$ then $(y, x) \in S$.

Is $(-3, 3)$ in S ?

Is $(168, -27)$ in S ?

Can we prove it?

Propositional Logic

Proposition / Formula

An statement that could
be argued as true or false

Do You Recognize These Symbols?

$$(CS\ 60 \vee CS\ 42 \vee CS\ 52)$$
$$\wedge$$
$$(\neg\text{Math}\ 55 \rightarrow CS\ 55)$$
$$\wedge$$
$$\neg CS\ 81.$$

Well-Formed Formulas

(an inductive definition!)

p, q, r , etc., are WFFs.

\top and \perp are WFFs.

If A is a WFF then so is $\neg A$.

If A and B are WFFs then so is $(A \wedge B)$.

If A and B are WFFs then so is $(A \vee B)$.

If A and B are WFFs then so is $(A \rightarrow B)$.