Regular Languages

October 31, 2011

CS 81: Computability & Logic

Recall

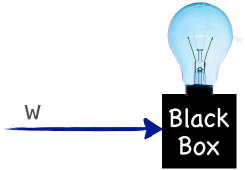
1. Computability questions reduce to *decision problems*.

2. Every decision problem can be rephrased as a question about membership in a *language*.

Determining What's Computable

The Plan:

- ✓ Define a class of abstract "machines" that accept or reject strings w



- ✓ See what languages this class of machines can recognize (i.e., what decision problems it can solve).

Note:

- ✓ Every machine corresponds to a language (its accepted strings)
- ✓ There may be many different machines accepting the same language
- ✓ It's unlikely that every language has a corresponding machine. (Why?)

One Possible Class of Machines

"When the term 'machine' is used in ordinary discourse, it tends to evoke an unattractive picture. It brings to mind a big, heavy, complicated object which is noisy, greasy, and metallic; performs jerky repetitive, and monotonous motions; and has sharp edges that may hurt one if he does not maintain sufficient distance…"

Marvin Minsky, *Computation: Finite and Infinite Machines*

But, can we generalize from this?

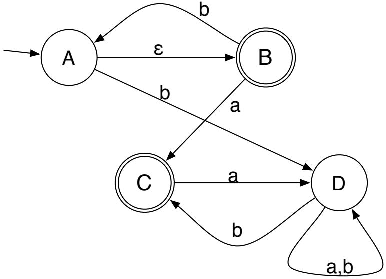
Our First Class of Machines: State Machines

Mathematically, a **state machine** consists of:

1. an alphabet Σ
2. a collection of states K
3. a transition relation $\rightarrow \quad \subseteq \quad K \times (\Sigma \cup \{\varepsilon\}) \times K$
   (where $q \overset{\sigma}{\rightarrow} q'$ means that (q, σ, q') is in the relation)
4. one initial/starting state $s \in K$.
5. a set of final/accepting states $A \subseteq K$.

finite state machine:
K is finite

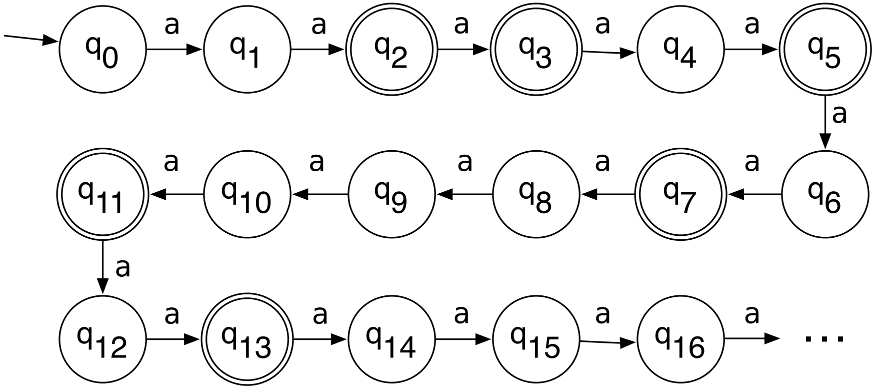
deterministic state machine:
a transition *function* $\delta : K \times \Sigma \rightarrow K$.



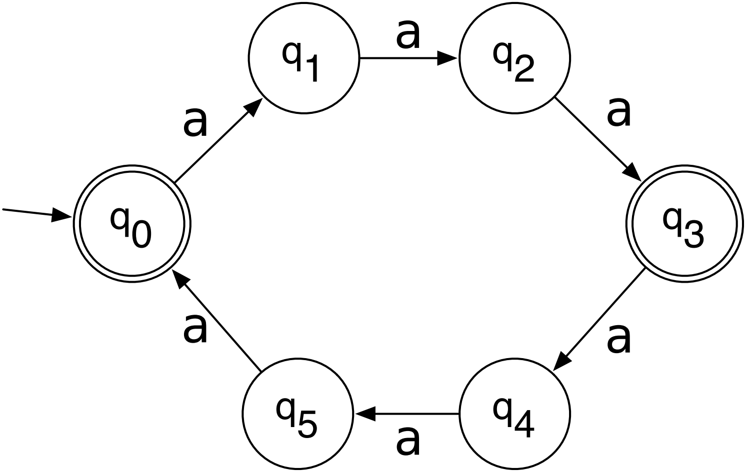
Machine Behavior

- ✓ The machine starts in state q_0.
- ✓ It can change from state q to state q' on input σ provided that $q \xrightarrow{\sigma} q'$.
- ✓ It can change from state q to state q' spontaneously provided that $q \xrightarrow{\varepsilon} q'$.
- ✓ The machine accepts a string $w \in \Sigma^*$ if there is *at least one* path spelling out w, that starts at q_0 and ends at a state $\in F$

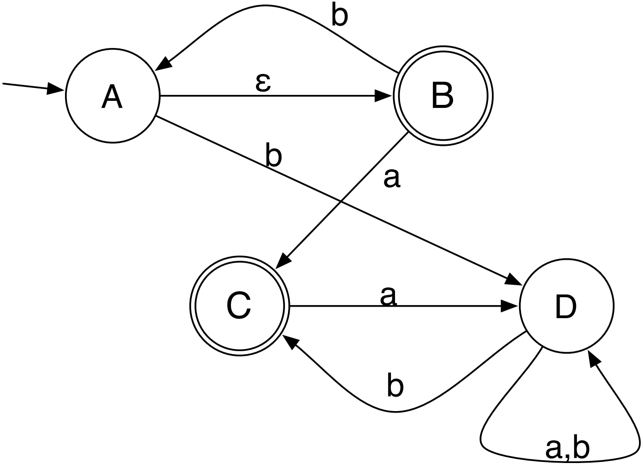
What's Accepted?



What's Accepted?



WHAT'S ACCEPTED? (bb? b? aaab? ba?)



Finite State Machines

We care mostly about *finite* state machines, also known as "Finite Automata"

Terminology:

- ✓ DFA = Deterministic Finite Automaton = Deterministic FSM = DFSM
- ✓ NFA = Nondeterministic Finite Automaton = Nondeterministic FSM = NDFSM

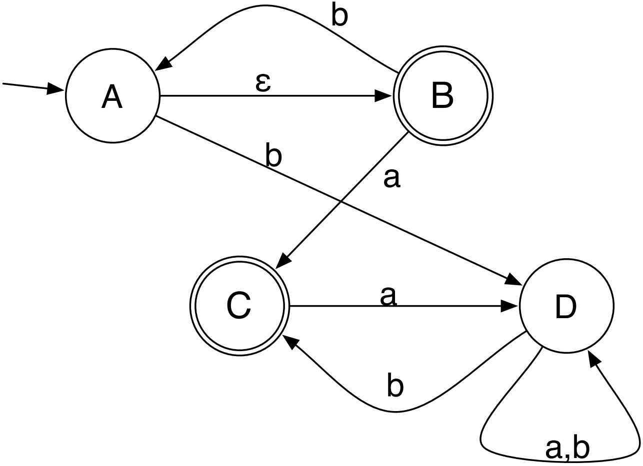
A Jewel of Theoretical Computer Science



The following are equivalent:

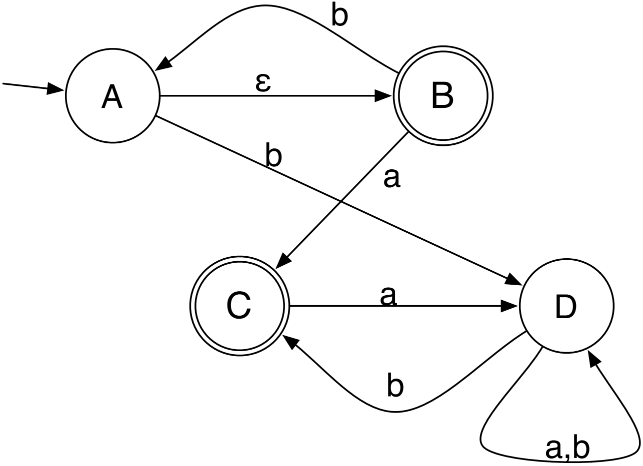
- ✓ There is a DFA accepting the language L
- ✓ [Rabin and Scott] There is an NFA accepting L
- ✓ [Kleene] L is a regular set.

Can a DFA predict this machine's behavior?



Digression: "Scotland Yard," the game

FROM NFA TO DFA: THE **SUBSET CONSTRUCTION**



Regular Languages

An *inductively-defined* collection of sets!

- ✓ $\emptyset$ is a regular language.
- ✓ $\{a\}$ is regular for any $a \in \Sigma$.
- ✓ If L and M are regular, then so is LM and $L \cup M$.
- ✓ If L is regular, then so is L^*.

True or False?

1. Σ^* is regular.
2. $\{\varepsilon\}$ is regular.
3. If $w \in \Sigma^*$, then $\{w\}$ is regular.
4. Every finite language is regular.
5. Every set is regular (since $\{w_1, w_2, \ldots\} = \{w_1\} \cup \{w_2\} \cup \cdots$).

Regular Expressions

An *inductively-defined* collection of expressions!

- ✓ $\emptyset$ is a regexp
- ✓ ε is a regexp
- ✓ a is a regexp for any $a \in \Sigma$.
- ✓ If r_1 and r_2 are regexps, then so is $(r_1 r_2)$ and $(r_1 | r_2)$.
- ✓ If r is a regexp, then so is (r^*).

Parenthesis Convention:

$$ab^*|c^* \quad = \quad (a(b^*)) \mid (c^*)$$

Regexp Interpretations

Regular expressions abbreviate regular languages.

- ✓ $L(\emptyset) = \emptyset$
- ✓ $L(\varepsilon) = \{\varepsilon\}$
- ✓ $L(a) = \{a\}$
- ✓ $L(r_1 r_2) = L(r_1)\, L(r_2)$
- ✓ $L(r_1 \mid r_2) = L(r_1) \cup L(r_2)$
- ✓ $L(r^*) = L(r)^*$

We say that "r matches w" if $w \in L(r)$. True or False?

- ✓ $L(r_1) = L(r_2) \quad \longrightarrow \quad r_1 = r_2$
- ✓ There is a regular expression r with $L(r) = \Sigma^*$

Regular Expression Examples ($\Sigma = \{0, 1\}$)

Describe the Language

1. $0 \mid 1$
2. $(0|1)^*$
3. $(0|1)\, 0^*\, 1^*$
4. $0^*110^*|1^*001^*$

Find the regular expression

1. Strings where *every* 1 is followed by a 0.
2. Strings where no 1 is followed by a 0.
3. Strings where *every* 1 is preceded by and followed by 0.