Performance Metrics

- Speedup
- Efficiency
- Work
- Scaled speedup

Speedup

- A problem is ideally solvable faster with multiple processors.
- Speedup =

$$\frac{\text{time to solve sequentially}}{\text{time to solve in parallel}}$$

- Some qualification is necessary:
  - Are the sequential and parallel times necessarily using the same algorithm?

Algorithm Dependence

- Not all algorithms parallelize equally well.
- Parallel execution may introduce overheads not present in sequential execution of a given algorithm.
- Rigorous definition of speedup demands that parallel execution of a given algorithm be compared against serial execution using the “best” sequential algorithm.
- Often this is not done: instead, the same algorithm is used for both sequential and parallel.

Speedup as a function of the number of processors

- Let T_p be the time required to solve a given problem using p processors.
- Let S_p be the speedup using p processors.
- $S_p = T_1 / T_p$
- Ideally $S_p = p$
- The ideal is difficult to achieve.

Amdahl's Law
(Gene Amdahl, 1967)

- Amdahl's law expresses a reason why ideal speedup may not be achieved.
- Make the idealized assumption that the code to be executed consists of
  - A perfectly-parallel portion (capable of using all p processors efficiently), and
  - A strictly-sequential portion (capable of using only 1 processor).
- Let **f** be the fraction of instructions that fall into the strictly-sequential category.

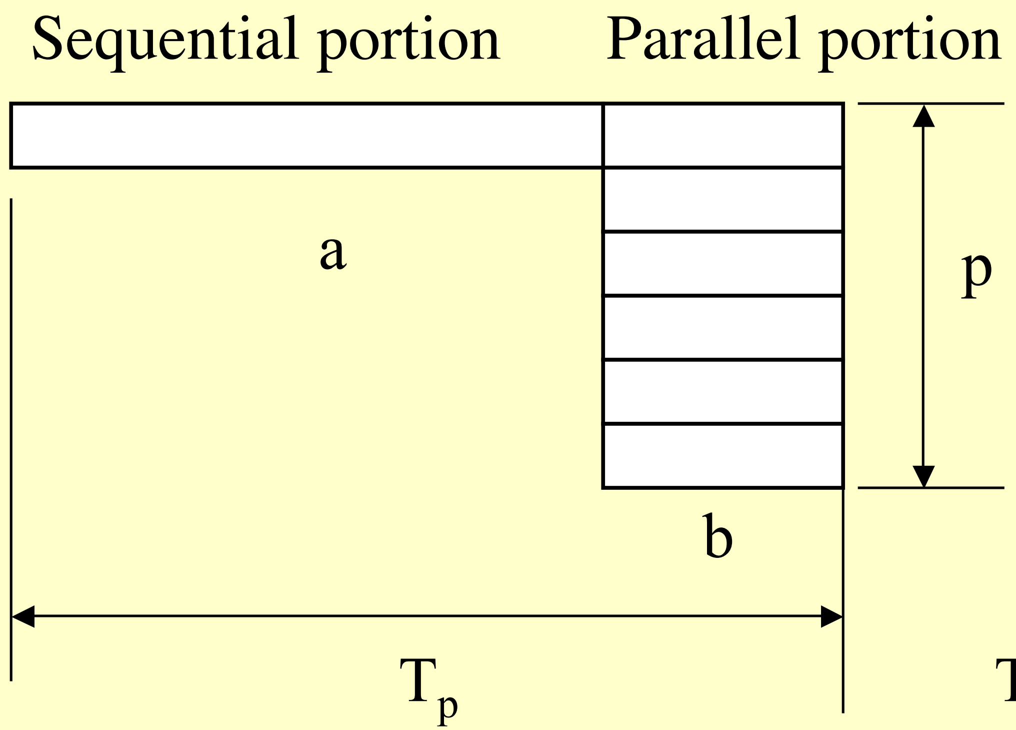
Amdahl's Law (2)

- Ideally, f is low. If f is 0, perfect speedup can be expected, while if f is high, speedup will be near 1.

- What is unexpected is how quickly speedup drops off as a function of f.

Amdahl's Law (3)

Program Execution Profile



$T_p = a + b$

$T_1 = a + pb$

Sequential fraction = f = a / (a + p*b)

Amdahl's Law (4)

Sequential fraction = f = a / (a + p*b)
Therefore f*(a + p*b) = a
Therefore b = (a/f - a)/p = (a/p)(1/f - 1)
Therefore b/a = (1/f - 1)/p

$$\begin{aligned}\text{Speedup} &= T_1 / T_p = (a+pb)/(a+b) \\ &= (1+pb/a)/(1+b/a) \\ &= (1 + (1/f - 1)/(1+(1/f-1)/p) \\ &= (1/f)/(1+(1/f-1)/p)\end{aligned}$$

$$\boxed{\text{Speedup} = 1/(f + (1-f)/p)}$$

Amdahl's Law, where f is sequential

Amdahl's Law (5)

Speedup $= 1/(f + (1-f)/p)$

Limiting cases and examples:

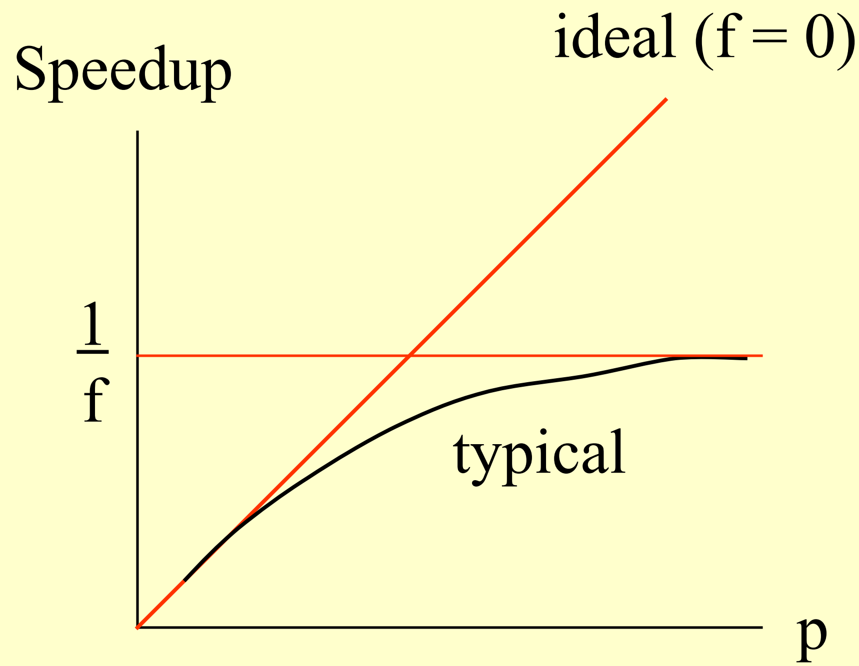
$p \rightarrow 1$: Speedup $\rightarrow 1$

$p \rightarrow \infty$: Speedup $\rightarrow 1/f$

$f \rightarrow 0$: Speedup $\rightarrow p$

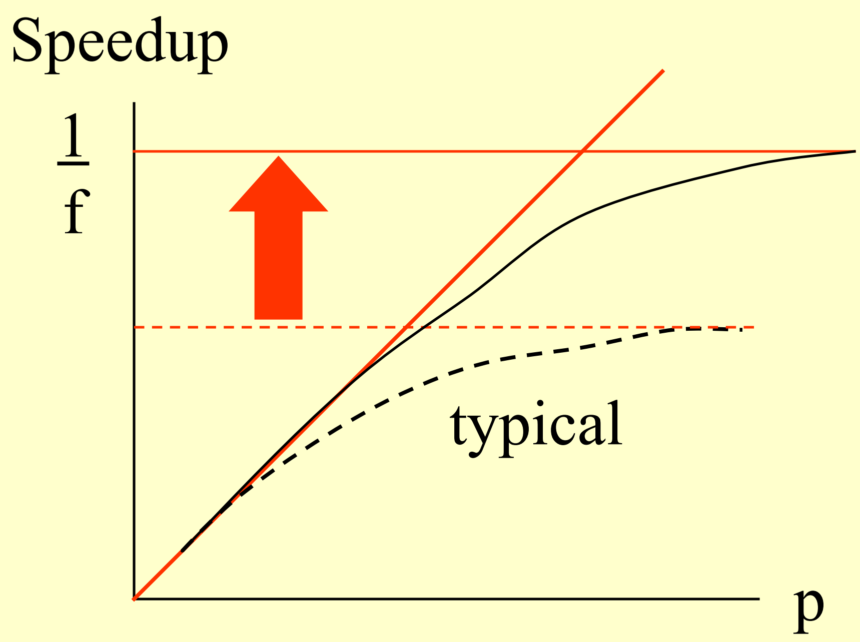
$f \rightarrow 1$: Speedup $\rightarrow 1$

$f = 0.1$: Speedup < 10



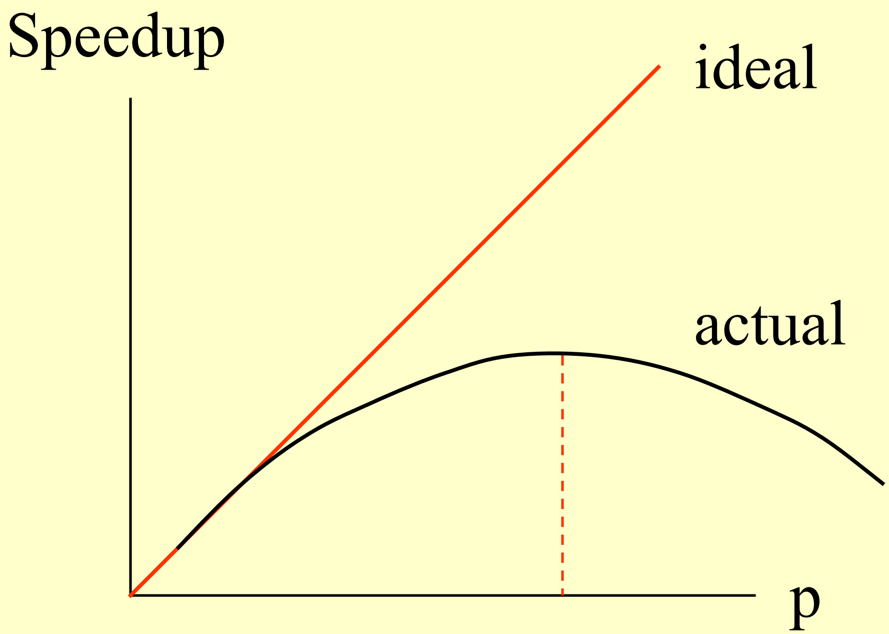
Amdahl's Law (6)

To lift the ceiling on speedup, we need to decrease f.



Slowdown

- Due to **overhead** of supporting more processors, speedup may actually *decrease* after a point:



Effort

- The effort used by a parallel processor in executing a program is the product of
  - the elapsed time, and
  - the number of processors used
- Included in effort are processors that are used for some portion of the computation, but which are idle for other portions.

Effort (2)

- Ideally the effort is the same regardless of the number of processors.
- In practice, the effort tends to go up with more processors, due to:
  - Overhead in spawning parallel processes
  - Communication overhead
  - Some processors being idle part of the time
- Usually we are willing to sacrifice some effort to attain speedup.

Efficiency

- How well are the parallel processors being utilized?
  - If there are p processors, with parallel execution time T_p, then the effort is p T_p.
  - The actual "work" done is T_1, the time it would take to do the work on one processor.
  - Therefore,

$$\text{Efficiency} = \frac{T_1}{p\,T_p}$$

  which happens to be equal to Speedup / p.

Ideal Efficiency

- The ideal efficiency is 1, with actual practice being somewhat worse, due to the additional effort mentioned earlier.
- We shouldn’t be lulled into thinking that efficiency is how busy the processors are, because they could be doing work that is parallel overhead.

Gustafson's "Law" (John Gustafson, 1988)

- Gustafson tried to refute Amdahl's law, which assumes that we are interested in applying ever larger numbers of processors to a fixed-sized problem.
- In practice, we are only interested in applying more processors as the size of the problem scales.
- Moreover, scaling the problem usually scales the parallel part disproportionately.

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Gustafson's Law (2)

- Let n be a measure of the problem size.
- The execution of the program on a *parallel* computer is decomposed into

  $$a(n) + b(n) = 1$$

  where a is the *sequential* fraction and b the *parallel* fraction (ignoring overhead for now).
- On a *sequential* computer, the relative time would be $a(n) + pb(n)$ where p is the number of processors in the parallel case.

Gustafson's Law (3)

- Speedup is therefore

  $(a(n) + pb(n))$ (relative to $a(n)+b(n) = 1$)

  $= a(n)+p*(1-a(n))$

  where $a(n)$ is the serial fraction.
- Assuming the serial fraction $a(n)$ diminishes with problem size n, then speedup approaches p as $n \to \infty$ as desired.
- Thus Gustafson's law seems to rescue parallel processing from Amdahl's law.

Granularity Considerations

- Roughly speaking, **granularity** means the ratio of computation interval to communication time needed to achieve a reasonable speedup.
- If a process needs to communicate frequently with other processes, then the communication must be very fast or the process' waiting time will absorb the speedup from parallel execution.

Granularity (2)

- Finer granularity is better, since it provides more ways to distribute the work.
- Imagine that the computation work load is a 10 kg. of material:
  - Sand = fine-grain
  - Cinder blocks (with or without warts) = coarse grain
- Which is easier to distribute?

Granularity (3)

- Fine-grain parallelism requires relatively-frequent communication compared to the computation interval.
- Consequently, fine-grain is more suited to shared memory than to distributed memory. Conversely, distributed memory requires relatively coarse grain to be effective.
- Because SIMD has less synchronization overhead, very-fine grain is more suited to SIMD than to MIMD.