Harvey Mudd College
Computer Science 81
Spring 2010

Assignment 1

Propositional Natural Deduction Proofs

Due. start of class, Wed., 26 January 2011

The problems in this set should be worked individually. You may get help, but transcribing solutions from another source is not allowed.

Prove each sequent using natural deduction. Use Fitch diagrams, except as indicated.

1. $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$
   [Hint: Use rules $\wedge$E, $\wedge$I, $\vee$E, $\vee$I, but $\vee$I is not usually the last step.]

2. $(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$

3. $\vdash ((p \to r) \vee (q \to r)) \to ((p \wedge q) \to r)$ [Empty set of hypotheses]

4. $(p \vee q) \to r \vdash (p \to r) \wedge (q \to r)$ [Use a tree diagram.]

5. $\neg p, \neg q \vdash \neg(p \vee q)$

6. $\neg(p \wedge q) \vdash \neg p \vee \neg q$ [Hint: Nested boxes and RAA may be needed.]

7. Recall that for sets, we have the following meanings:
   - $A \subseteq B$ means: (for arbitrary x) $x \in A \to x \in B$
   - $C = A \cup B$ means: (for arbitrary x) $x \in C \leftrightarrow (x \in A \vee x \in B)$
   - $C = A \cap B$ means: (for arbitrary x) $x \in C \leftrightarrow (x \in A \wedge x \in B)$

   Use natural deduction to show:

   $$(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$$

8-10. Show that, given only **one** of the rules RAA (reductio ad absurdum), LEM (law of the excluded middle), DNE (double not elimination), the other two rules can be derived using only intuitionistic rules in addition. [This need entail only three derivations: LEM from RAA, DNE from LEM, and RAA from DNE, for example.]