

CS81 Assignment 2

Due Wednesday, 2 February 2011

1. If the formula $(p \rightarrow q) \vee (q \rightarrow r)$ is a tautology, provide a natural deduction proof of it. If not, give a counterexample valuation.
2. Determine whether or not the following entailment is valid: $(q \rightarrow (p \vee (q \rightarrow p))) \vee \neg(p \rightarrow q) \models p$
3. Suppose that Γ and Δ are sets of propositional formulas. For each statement below, if the statement is true, prove it. If it is false, give a counter-example.
 - a. If Γ is satisfiable and $\Gamma \subseteq \Delta$, then Δ is satisfiable.
 - b. If Γ is satisfiable and $\Delta \subseteq \Gamma$, then Δ is satisfiable.
4. Show that $\Gamma \models \psi$ iff $\Gamma \cup \{\neg\psi\} \models \perp$. To what natural deduction rule(s) is this a counterpart?
5. Give a natural deduction proof *in contextual form* for $\vdash ((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$
6. Prove the part of the induction step of the **soundness theorem**, as discussed in the lecture slides, for the case of the \vee **Elimination rule**.
7. For the tautology $(p \rightarrow q) \vee (q \rightarrow p)$, show the proof that would be constructed in the structural induction part of the proof of the **completeness theorem** given in the lecture slides.
8. Use mathematical induction to prove statements 1 and 3 on slide 33 of the lecture (completeness proof).