

CS81 Assignment 5
Due Wednesday, 23 February 2011

1. [20] Let $L(x, y)$ mean “ x loves y ”. Translate each of the following statements into a predicate logic formula. John, Mary, and Tom are constant symbols.
- a. John loves Mary.
 - b. Mary loves herself.
 - c. John is a lover (meaning there is someone John loves).
 - d. John loves everyone except Mary.
 - e. John loves exactly one person.
 - f. Mary loves exactly two people.
 - g. Tom loves no one.
 - h. Tom loves anyone who does not love him.
 - i. Tom loves anyone who does not love him/herself. (Tom may love others as well.)
 - j. Tom loves any and only those who do not love him/herself.

In 2-6, check each syllogism or sequent for validity by the tableau method. If not valid, give a counterexample. Assume that “a lover” means a person who loves someone. That is, if $L(x, y)$ means “ x loves y ”, then “ x is a lover” is the same as $\exists y L(x, y)$.

2. [2] Given
- a. Everyone loves a lover.
 - b. Mary loves herself.

It follows that

- c. Mary loves John.

3. [3] Given
- a. Tom loves anyone who does not love him/herself.

It follows that

- b. Tom loves himself.

4. [5] Given
- a. Tom loves any and only those who do not love him/herself.

It follows that

- b. Tom loves Mary.

5. [10] $\vdash ((\forall x A(x)) \rightarrow (\exists x B(x))) \rightarrow (\forall x (A(x) \rightarrow B(x)))$

6. [10] $((\forall x A(x)) \rightarrow (\exists x B(x))) \vdash (\exists x (A(x) \rightarrow B(x)))$

(continued)

In 7-8, prove by the Sequent Calculus, or give a counterexample.

7. [10] $(\exists x (A(x) \rightarrow B(x))) \mid\text{---} ((\forall x A(x)) \rightarrow (\exists x B(x)))$

8. [10] $(\forall x A(x)) \rightarrow \exists x B(x) \mid\text{---} \exists x (A(x) \rightarrow B(x))$

9. [30] (Huth&Ryan, exercise 4.6.5) Use Hoare logic to prove the partial correctness of the following division program. All variables are of type integer.

$\{y > 0\}$ assumption

```
r = x;
d = 0;
while( r >= y)
{
    r = r - y;
    d = d + 1;
}
```

$\{x = d*y + r \wedge r < y\}$ expectation

Start by finding an appropriate loop invariant. Show the triples as a numbered list, with justifications for each. Also show the assertions as inline comments in the program. Tabulate any assumptions you are making about integer arithmetic.