Assignment 8: Regular and Nonregular Languages

Due: Wednesday, April 6

- Emails about this assignment should be directed to cs81help@cs.hmc.edu .
- Diagrams may be hand-drawn or computer-drawn via a program of your choice. If you use special-purpose tools such as JAPE to draw automata, do not use it to solve the problems. (After all, you won't be able to use JAPE during the final exam!) All work submitted must be your own.
- Make sure your submission includes your name!

1. Read pp. 77–82 of Sipser. Come up with (at least) one question about the reading where you're not sure of the answer. These may relate to points where the book is confusing, or simply to some related question or conjecture that occurs to you while doing the reading.

2. Do Exercise 1.4(f) on page 83.

3. Provide clear and convincing proofs for Problem 1.42, page 89. (The notation in the book can be easily misinterpreted: $a_1 \cdots a_k$ is a *single* word in A, and the a_i's are consecutive pieces ("substrings") of that word. Also, any of the a_i's might be the empty string. So for example, strings in the shuffle of $\mathtt{a}^+$ and $\mathtt{b}^+$ include abab, aabbaabb, aabbaa, ba, babbabbaaa, and so on. The intuition is to take a single string from A and a single string from B and do the equivalent of one random "riffle shuffle" of two piles of cards.)

4. Define
$$\text{Prefix}(L) := \{\, w \in \Sigma^* \mid \exists z \in \Sigma^*.\, wz \in L \,\}.$$
Intuitively, w is in $\text{Prefix}(L)$ if and only if it is a prefix of some word in L. Prove that if L is a regular language then $\text{Prefix}(L)$ is also regular.

5. Do Problem 1.32, page 88

6. Do Problem 1.33, page 89

7. Do Problem 1.34, page 89

8. Do Problem 1.53, page 91.

9. Here are a few languages with $\Sigma = \{0, 1\}$. Decide whether each is regular or not, and prove your answer is correct.

   (a) The set of strings having the property that in every prefix, the the number of 0's and the number of 1's differ by no more than 2.

   (b) The set of strings s for which there exists an integer $k > 1$ (possibly depending on s) such that the number of 0's in s and the number of 1's in s are both divisible by k.

   (c) The set of all strings x having some non-empty substring of the form www, e.g., 111 and 00101011. (You may make use of the following true fact: the set of strings in Σ^* that *do not* contain a non-empty substring of the form www is infinite, and hence contains strings as long as you'd like.)

10. You know several ways of specifying regular languages, including regular expressions, DFAs, and NFAs. Assuming that regular languages might be provided to you in any of these forms, carefully describe strategies for solving the following problems.

    You will probably want to use previously-seen algorithms as building blocks (the subset construction, DFA minimization, etc.)

    (a) Given a regular language L, decide whether the empty string is in L.

    (b) Given a regular language L, decide whether it is empty or not.

    (c) Given a regular language L, decide whether it is finite or infinite.

    (d) Given regular languages L_1 and L_2, decide whether they have at least one string in common.

    (e) Given regular languages L_1 and L_2, decide whether $L_1 = L_2$.

    (f) Given regular languages L_1 and L_2, decide whether $L_1 \subseteq L_2$.