Computer Science 81, Spring 2011
Resolution Practice Problems
Work before the mid-term, but do not submit.

Checking your solutions with Otter-lambda or Prover9 is advisable.

1. [10 points] Establish the satisfiability or unsatisfiability of the following sets of clauses by the resolution method. If a set is satisfiable, give an assignment that shows this.
   a. $\{p \vee q,\ p \vee \neg r,\ \neg p \vee r,\ \neg p \vee \neg q,\ \neg q \vee r,\ q \vee \neg r\}$

   b. $\{p \vee q, \neg p \vee r,\ \neg p \vee \neg q,\ \neg q \vee r,\ q \vee \neg r\}$

2. [10 points] Determine whether each pair of terms is unifiable, and if so, give the most general unifier. Note: u, v, w, x, y, z are variables; a, b, c, d are constants.

   a. $p(f(x, g(y)), y)$ *vs*. $p(f(g(a), z), b)$

   b. $p(x, f(y, a), y)$ *vs*. $p(f(a, b), v, z)$

   c. $p(x, f(x))$ *vs*. $p(g(a), f(h(a)))$

3. [20 points] Establish the satisfiability or unsatisfiability of the following sets of clauses by resolution. If a set is satisfiable, give an interpretation that shows this.

   a. $\{p(x),\ q(y, a) \vee \neg p(y),\ \neg q(b, a)\}$

   b. $\{p(a),\ q(y, a) \vee \neg p(a),\ q(b, x)\}$

   c. $\{q(x),\ \neg p(y) \vee \neg p(g(a)) \vee \neg q(a),\ p(z) \vee \neg q(w)\}$

4. [20 points] Transform each formula into an equivalent set of clauses.

   a. $\exists y\, \forall x\, (p(x, y) \rightarrow q(x))$

   b. $\forall x\, \exists y\, (p(x, y) \vee \exists z\, q(x, y, z))$

5. [40 points] Prove each formula by the resolution method:

   a. $\forall x\, (p(x) \rightarrow q(x)) \rightarrow ((\forall x\, p(x)) \rightarrow (\forall x\, q(x)))$

   b. $\forall x\, (p(x) \rightarrow q(x)) \rightarrow ((\exists x\, p(x)) \rightarrow (\exists x\, q(x)))$

   c. $\exists x\, (p(x) \rightarrow \forall x\, p(x))$

   d. $((\exists x\, p(x)) \rightarrow (\forall x\, q(x))) \rightarrow (\forall x\, (p(x) \rightarrow q(x)))$