Environments and Bindings

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October 2012

What is a Binding?

- A ***binding*** is an association between an identifier (or variable) and a value.
- We say the identifier is "bound to" the value.

- Examples:

  X is bound to 5

  foo is bound to "bar"

  null is bound to OpenList.nil

What is an Environment?

- An environment is a set of bindings.

Why is this important?

- Environments change in going from one part of an expression to another.
- For the most part, such changes can be accomplished non-destructively.
- Old environments are not destroyed, just modified by layering on new bindings.

How is an environment represented?

- There are multiple ways, but in the Rockit interpreter, we use an association list.
- '((X 5) (Y 10) (Z 15) (X 20))

  X is bound to 5

  Y is bound to 10

  Z is bound to 15

  The remaining X has no effect, as association lists are searched front to back.

Changing Environments

- These forms all have a *temporary* effect on the environment:

  lambda

  let

  let*

  letrec

Example1: let

- Say the ambient environment is

  ‘((X 5) (Y 10) (Z 15) (X 20))

  Consider the expression

  (let ((Y 30)) (+ X Y Z))

Example1: let

- Say the ambient environment is

  ‘((X 5) (Y 10) (Z 15))

  Consider the expression

  (let ((Y 30)) (+ X Y Z))

overridden binding

‘((Y 30)(X 5) (Y 10) (Z 15))

body environment

‘((X 5) (Y 10) (Z 15))

outer environment

Example2: nested lets

Consider the expression

(let ((Y 30)) (let ((Z 0)) (+ X Y Z)))

‘((X 5) (Y 10) (Z 15))

outer environment

Example2: nested lets

Consider the expression

(let ((Y 30)) (let ((Z 0)) (+ X Y Z)))

‘((Y 30)(X 5) (Y 10) (Z 15))

mezzanine body environment

‘((X 5) (Y 10) (Z 15))

outer environment

Example2: nested lets

Consider the expression

(let ((Y 30)) (let ((Z 0)) (+ X Y Z)))

‘((Z 0)(Y 30)(X 5) (Y 10) (Z

innermost body environment

‘((Y 30)(X 5) (Y 10) (Z 15))

mezzanine body environment

‘((X 5) (Y 10) (Z 15))

outer environment

Example 3: Variables in right-hand sides of equations

- The variable bindings defined by let expressions can be viewed as determined from “equations”:

(let ((Y 30)) …) is like Y = 30

The right-hand sides can contain variables

(let ((Y (* Z Z))) …) is like Y = (* Z Z)

Example 3: Variables in right-hand sides of equations

- This raises the question of “In what environment are the right-hand sides evaluated?”

- The answer is: in the environment containing the let expression.

Example 3: nested lets

Consider the expression

(let ((Y (+ X 2))) (let ((Z (+ X Y))) Z))

‘((X 5) (Y 10) (Z 15))

outer environment

Example 3: nested lets

Consider the expression

(let ((Y (+ X 2))) (let ((Z (+ X Y))) Z))

‘((Y 7)(X 5) (Y 10) (Z 15))

mezzanine body environment

‘((X 5) (Y 10) (Z 15))

outer environment

Example 3: nested lets

Consider the expression

(let ((Y (+ X 2))) (let ((Z (+ X Y))) Z))

‘((Z 12)(Y 7)(X 5) (Y 10) (

innermost body environment

‘((Y 7)(X 5) (Y 10) (Z 15))

mezzanine body environment

‘((X 5) (Y 10) (Z 15))

outer environment

Example 4: “Redefining” lets

- (let ((X (* X X))) X)

‘((X 25) (X 5))

inner environment

‘((X 5)

outer environment

Layering

- When interpreting a let expression, it is not necessary to change the outer environment.
- Instead, just lay on new bindings for evaluating the body of the let.
- When evaluation is complete, **the original environment will still be intact**.

Let* vs. Let

- let* is like a cascade of nested lets:
- (let* ((X 5) (Y (* X X)) (X (+ X Y)) …)

  is equivalent to

```
(let ((X 5))
        (let ((Y (* X X)))
              (let ((X (+ X Y)))
                    …
              )
        )
)
```

Letrec

- In letrec, the environment for the RHS is the same as the inner environment.
- (letrec ((f (lambda(x)

```
            (if (< x 2) 1 (* x (f (- x 1)))))))
   (f 5)
)
```

  works, but will not if letrec is replaced with let.

  Why?

letrec vs. let

```
> (letrec ((f (lambda(x) (if (< x 2) 1 (* x (f (- x 1))))))) (f 10))
3628800

> (let ((f (lambda(x) (if (< x 2) 1 (* x (f (- x 1))))))) (f 10))
. . f: undefined;
 cannot reference an identifier before its definition
```

Using *define*

- Will this work?

```
> (define f (lambda(x) (if (< x 2) 1 (* x (f (- x 1))))))

> (f 10)
```

The base environment

- *define*, at the top level, adds bindings to the base environment.

- The base environment **is used if no binding exists otherwise**.

```
> (define f (lambda(x) (if (< x 2) 1 (* x (f (- x 1))))))
> (f 10)
3628800
```

In Rockit Interpreter

- In the variable lookup part of eval, have it check the base environment after checking the environment passed to eval.

```
public static Object eval(Object ob, OpenList env)
  {
  if( ob instanceof OpenList )
      {
        // An OpenList expression is some kind of form
        return evalForm((OpenList)ob, env);
      }

    if( ob instanceof String )
      {
        // A String is a variable
        OpenList found = env.assoc(ob);

        if( found.nonEmpty() )
          {
            return found.second();
          }

        // Checking the base environment after the environment arg enables
        // recursion in a natural way.

        found = baseEnvironment.assoc(ob);

        if( found.nonEmpty() )
          {
            return found.second();
          }
```

Lambda Expressions

- Lambda expressions translate into function objects (usually called "closures") that carry their environment with them.
- Sometimes this matters, sometimes not (when?)
- The base environment is still checked if a binding is not found.

Bindings for Lambda Expressions

(lambda (X) (+ X Y))

overridden binding

‘((X 6) (Y 5))

ambient environment (for example)

‘((X ??) (X 6) (Y 5))

body environment

The body environment is only known when the function is applied.

Bindings for Lambda Expressions

binding

((lambda (X) (+ X Y)) 99)

application (of function to 99)

‘((X 6) (Y 5))

ambient environment

‘((X 99) (X 6) (Y 5))

body environment

The body environment is only known when the function is applied.

How to Bind Lambdas

If there is a keyword ***lambda*** immediately inside the opening left paren, the expression represents a function.
Otherwise it is an **application**.

Start from the outermost application that can be evaluated and work inward, adding binding layers as you go.

Bindings can also be represented by *replacing* free variables with their values.

Constructing/Deconstructing Examples

- (lambda (X) …) function
- ((lambda (X) …) 4) application
- If the above function returns a function, then (((lambda (X) …) 4) 5) is that function applied.

Example: function returning a function

- (((lambda (X) (lambda(Y) (list X Y)) 4) 5)
  ((lambda(Y) (list 4 Y)) 5)
  (list 4 5)
  [This is the “Curried” version of the list function.]

Similarity is not Identity

(((lambda (X) (lambda(Y) (list X Y)) 4) 5) before
vs.
((lambda(X) ((lambda(Y) (list X Y)) 4)) 5) now

In the second case, the body of the outermost function is itself the ***result*** of an application: ((lambda(Y) (list 5 Y)) 4)), but not a function itself.

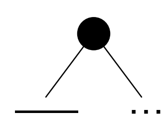
Gives ‘(5 4), not ‘(4 5) as before.

Tree Representation

- When all else fails, expressions can be parsed as a **tree**.

(lambda (X) …) function

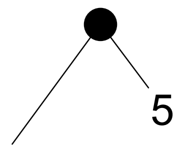
(___ …) application



Applies a function ___ to ….

Tree Example

- (((lambda (X) (lambda(Y) (list X Y)) 4) 5)

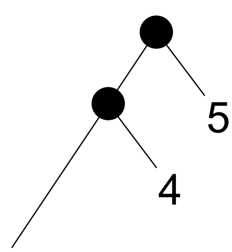


((lambda (X) (lambda(Y) (list X Y)) 4)

application, or function?

Tree Example

- (((lambda (X) (lambda(Y) (list X Y)) 4) 5)

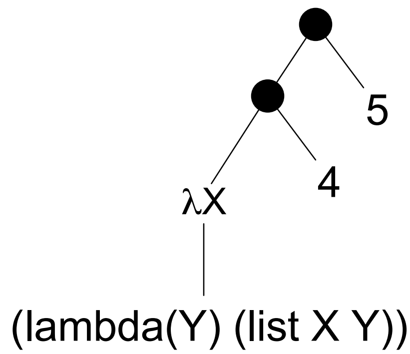


(lambda (X) (lambda(Y) (list X Y))

application, or lambda exp?

Tree Example

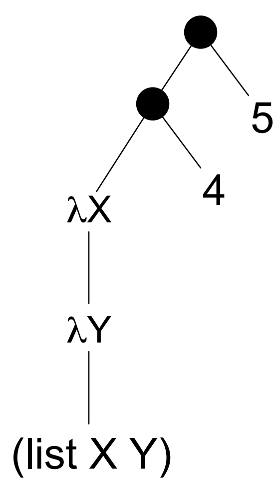
- (((lambda (X) (lambda(Y) (list X Y)) 4) 5)



application, or lambda exp?

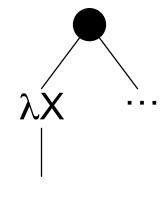
Tree Example

- (((lambda (X) (lambda(Y) (list X Y)) 4) 5)



Evaluating

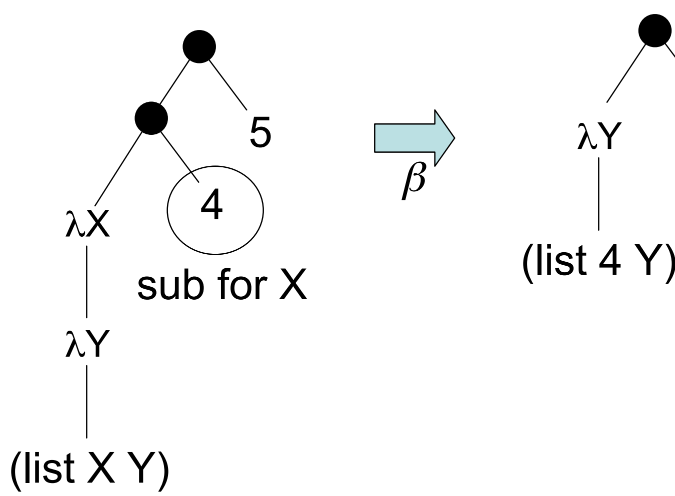
- Application **annihilates** lambda



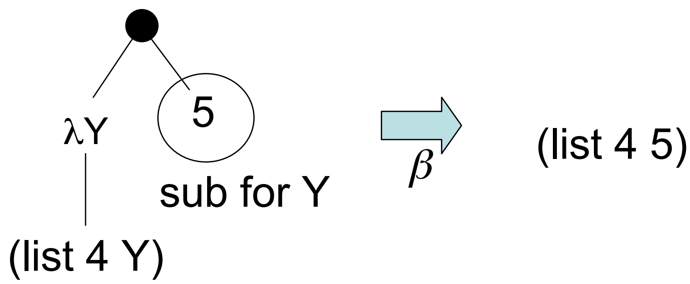
Substitute … for all ***free*** occurrences of X in ___.

[This step is called ***β reduction*** in CS & Logic.]

Evaluating



Evaluating



Example

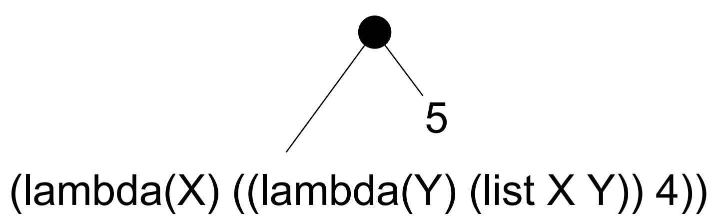
- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)
- Construct tree, then evaluate.

Example

- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)
  Application or function?

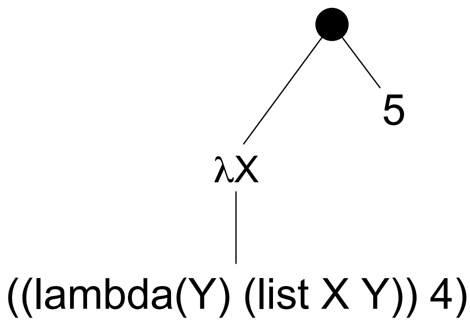
Example

- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)



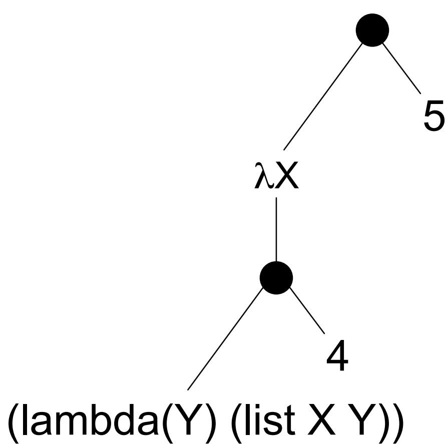
Example

- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)



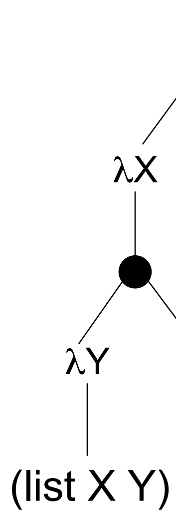
Example

- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)

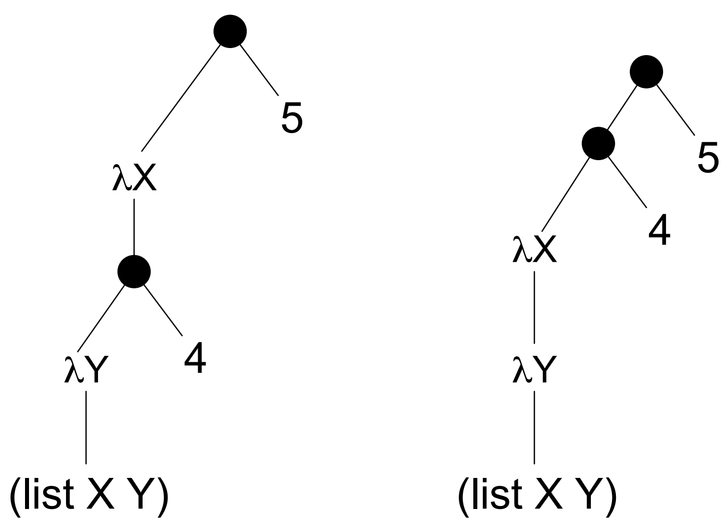


Example

- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)

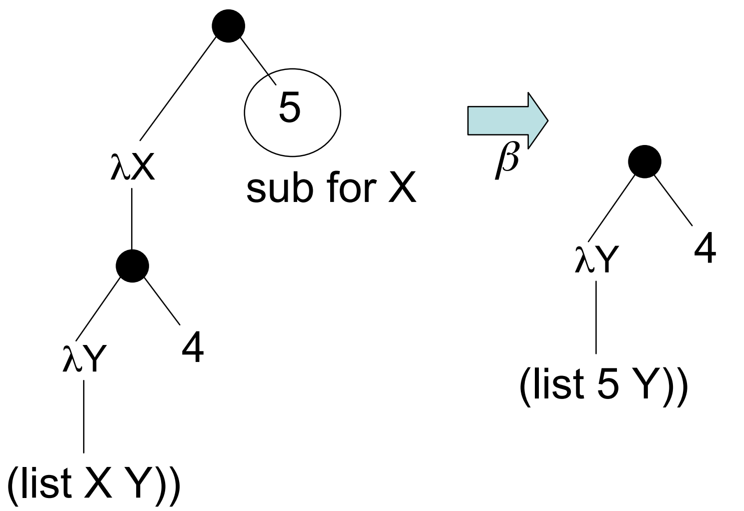


Two Trees Compared



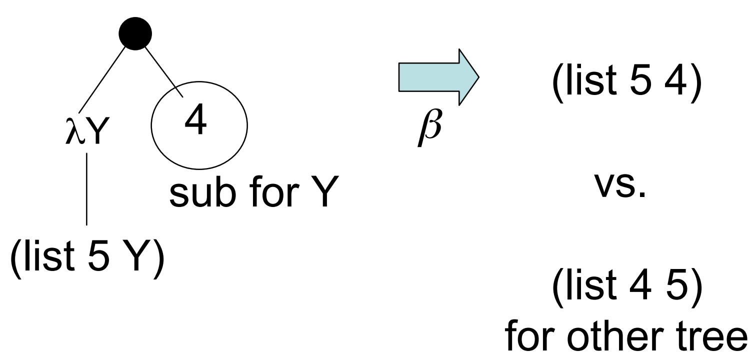
Example

- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)



Example

- ((lambda(X) ((lambda(Y) (list X Y)) 4)) 5)



Static vs. Dynamic Binding

- **Mathematical functions demand static binding**

```
(define y 99)
(define f (lambda(x) (+ x y)))
(let ((y 1000)) (f 1))
```

y in (+ x y) should be bound to 99 (**static** binding)

If it were **dynamic binding**, y would be bound to 1000 in (+ x y).

Static Binding Example 1

```
(let ((X 4))
   (lambda(Y) (list X Y)) 5)
 )
```

The application applies a function (lambda(Y) (list X Y))), with the static binding (X 4).

This function applied to 5 gives ‘(4 5).

Static Binding Example 2

```
(let ((X 4))
   (let ((f (lambda(Y) (list X Y))))
        (f 5)
   )
)
```

Same deal: The application applies a function
(lambda(Y) (list X Y))),
with the static binding (X 4).

This function applied to 5 gives ‘(4 5).

Static Binding Example 3

```
(let ((X 4))
   (let ((f (lambda(Y) (list X Y))))
        (let ((X 99))
                (f 5)
         )
   )
)
```

Still same deal: The application applies a function
(lambda(Y) (list X Y))),
with the static binding (X 4).

This function applied to 5 gives ‘(4 5).

Dynmamic Binding Example 1

```
(let ((X 4))
   (lambda(Y) (list X Y)) 5)
 )
```

The application applies a function ***expression***
(lambda(Y) (list X Y))).

X is bound to 4 in the environment of the application, so that is used.

This function applied to 5 gives ‘(4 5).

Dynamic Binding Example 2

```
(let ((X 4))
   (let ((f (lambda(Y) (list X Y))))
        (f 5)
   )
)
```

Same deal: The application applies a function expression
(lambda(Y) (list X Y)))

X is bound to 4 in the environment of the application, so that is used.

This function applied to 5 gives ‘(4 5).

Dynamic Binding Example 3

```
(let ((X 4))
   (let ((f (lambda(Y) (list X Y))))
        (let ((X 99))
                (f 5)
        )
   )
)
```

Different: The application is a function expression.
(lambda(Y) (list X Y))),
The application environment has X bound to 99.

The result is '(99 5), not '(4 5) the mathematical result.

Quasi-Static bindings

- In Racket, **bindings** in the **base environment are** statically bound to **locations (variables)** rather than values.

```
> (define x 99)
> (define g (lambda(y) (+ x y)))
> (g 1)
100

> (set! x 100)          ; destructively modifies the x value
> (g 1)
101                     ; The location hasn't changed,
                        ; but the value has.
```

Let as Lambda

- Lambda is more general than Let

- (let ((V1 E1) (V2 E2) … (Vn En)) E0)

  =

  ((lambda (V1 V2 … Vn) E0)
  E1 E2 … En)

  However, the static binding issue does not arise with let.

Double Example

- (define double (lambda (f) (lambda (x) (f (f x)))))

- Draw the tree

- Evaluate ((double square) 5)

- Evaluate (((double double) square) 5)

(double double)

```
> (double double)
#<procedure>

> (((double double) square) 5)
152587890625

> (square (square (square (square 5))))
152587890625
```

How Big?

```
(((double (double double)) square) 5)
```

How Big?

shot from my screen at 7-point font:

```
> (/ (log (((double (double double)) square) 5)) (log 10))
45,807 ; decimal digits
```

Composing Functions Using Functions

```
> (define (compose f g) (lambda (x) (f (g x))))

> (define (cube x) (* x x x))
> ((compose cube square) 5)
15625

> ((compose square cube) 5)
15625

> (define (add2 n) (+ 2 n))
> ((compose add2 square) 5)
27

> ((compose square add2) 5)
49
```

Draw a picture