Languages, Grammars and Parsing

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September 2012

What are These?

- **Language:** A set of strings.
- **Grammar**: A formal way to define a language.
- **Parser**: A program that determines whether or not a given string is in the language.

More Detail on Languages

- A language is a **set of strings** of symbols from a finite set called the **alphabet**.
- When we show languages, we usually don't show the strings with quotes, as a convenience.

Example 1

The language of all U.S. zipcodes

{00501, …, 91711, …, 99950}

alphabet = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

This language is finite.

Example 10

The language of all binary numerals without unnecessary leading zeros

{0, 1, 10, 11, 100, 101, 110, 111, …}

alphabet = {0, 1}

This language is infinite.

Example 11

- The language of all “2-adic” numerals

{ε, 1, 2, 11, 12, 21, 22, 111, 112, 121, …}

ε stands for the ***empty string***

alphabet = {1, 2}

This language is infinite.

Example Scrabble

The language of official Tournament Scrabble words =

{aa, aah, aahed, …, zyzzyvas, zzz}

alphabet = {a, b, c, …, z}

This language is finite.

Example WBP

- WBP = Well-Balanced Parenthesis Strings
- { (), (()), (()()), ((())), (()()()), (()()(())), … }
- alphabet = { (,) }
- This language is

Grammars

- A grammar is a **formal** (i.e. mechanical)way of defining a language.
- It is a special kind of **inductive definition**, something we've seen before
(think S expressions).
- It can also be viewed as a **state-transition system,** also déjà vu
(think Picobot).
- It is also a form of **rewriting system**, analogous to evaluating a function.

A Grammar Has 4 Parts

- An alphabet Σ, called the *terminal* alphabet.
- An alphabet N, called the *non-terminal* alphabet.
N does not overlap with Σ.
- A finite set of *rules* (also called "productions").
- A *start symbol*, always a member of N.

Rules

- A rule indicates how a non-terminal symbol within a string can be "rewritten", i.e. be **replaced by** a string.
- Only non-terminal symbols can be rewritten.
- Suppose there is a rule:

  $S \rightarrow (L)$

  This is read "S rewrites as (L)", or "S produces (L)".

Non-Determinism

- We have choices.
- For a given symbol, we may have more than one rule with that symbol as the left-hand side, e.g.

  $L \rightarrow A\ L$

  $L \rightarrow \varepsilon$ where ε is the empty string
- Grammars are easy if you follow the rules.

Grammar as State-Transitions

- A grammar defines a state-transition system.
- The **states** are strings from the combined alphabet $\Sigma \cup N$.
- The **transitions** between states are defined on the next page.
- The **initial state** is the start symbol, say S.

Transitions

One state, a string

$$\ldots X \ldots$$

containing a non-terminal X, can make a **transition** to another state,

$$\ldots \alpha \ldots$$

where α is a string of symbols in $\Sigma \cup N$ exactly when there is a **rule**

$$X \rightarrow \alpha \qquad \text{(rule)}$$

When this is possible, we write

$$\ldots X \ldots \Rightarrow \ldots \alpha \ldots \qquad \text{(transition)}$$

Example

- Suppose the rules are

  $S \rightarrow (L)$

  $L \rightarrow S\ L$

  $L \rightarrow \varepsilon$

- Then here are some possible transitions

$(L) \Rightarrow (S\ L)$	using rule $L \rightarrow S\ L$
$(L) \Rightarrow ( )$	using rule $L \rightarrow \varepsilon$
$(S\ L) \Rightarrow (S\ S\ L)$	using rule $L \rightarrow S\ L$
$(S\ L) \Rightarrow (S)$	using rule $L \rightarrow \varepsilon$
$(S\ L) \Rightarrow ((L)\ L)$	using rule $S \rightarrow (L)$

The Yield of a Grammar

- The **yield** of a grammar is the set of all states reachable from the start symbol by applying 0 or more rules in sequence.

- This can be done systematically by considering **each candidate rule** in turn for **each possible lefthand side symbol** in a state, beginning with the string consisting of just the start symbol.

Yield Example

- Consider the alphabets used previously, start symbol S, and rules

1	S → (L)
2	L → ε
3	L → S L

- The first part of the yield is

step	string	from state	using rule
0.	S	start symbol	N/A
1.	(L)	0	1
2.	( )	1	2
3.	(S L)	1	3
4.	(S)	3	2

Quiz

- Add the next 4 lines to the yield table.

1	S → (L)
2	L → ε
3	L → S L

step	string	from state	using rule
3.	(S L)	1	3
4.	(S)	3	2
5.			
6.			
7.			
8.			

Grammar to Language

- Given a grammar, the **language generated by** the grammar is subset of the yield containing only terminal symbols. We can construct the language by constructing the yield, collecting only the terminal strings as we go.

- The language generated by the previous grammar contains

  ( )
  (( ))
  (( ) ( ))
  ((( )) ( ))
  ((( )) (( )))
  etc.

Parsing

- Parsing addresses the problem:

  Based on a grammar G:

  Given a string x, is x in the language generated by G, or not?

  $x \in L(G)$?

- There is an obvious algorithm for parsing (what?) but it is generally too slow for practical use.

Determinism in Parsing

- Although we described grammars as forming a generally non-deterministic system, we want parsing to be as **deterministic** as possible.
- Ideally the time taken is O(n) where n is the length of the string x.

Parsing by Recursive Descent

- If the grammar is constructed the right way, there is an easy way to parse its language, using recursion.
- Illustrate with the previous grammar:

1	S → (L)
2	L → ε
3	L → S L

Parsing an Input String

- Given an input string, such as
  (( ) (( ) ( )))
  we want to know "is this string in the language?"

  In other words, is there a series of states of the form
  $S \Rightarrow \ldots \Rightarrow \ldots \Rightarrow \ldots \Rightarrow$ (( ) (( ) ( )))

  [abbreviated $S \Rightarrow^*$ (( ) (( ) ( )))]

  To answer such questions, **use the grammar** to construct a set of **parse functions**.

Parse Functions

- There will be one parse function parse-X for each non-terminal X.

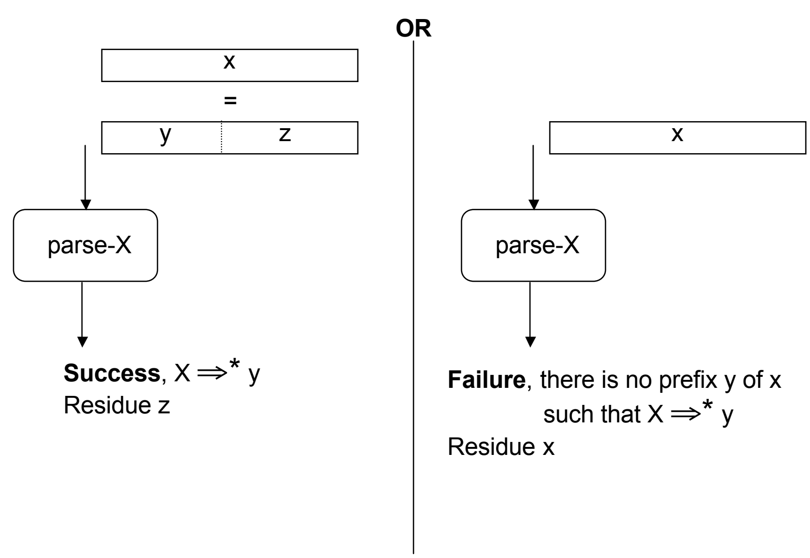
- For this grammar, we will have two functions:
  parse-S
  parse-L

1	$S \rightarrow (L)$
2	$L \rightarrow \varepsilon$
3	$L \rightarrow S\ L$

Parse Function Jobs

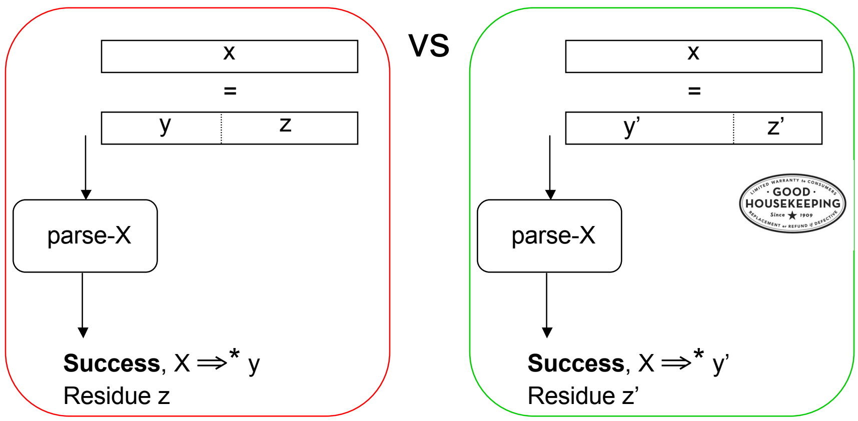
- On each call, a parse function has a string x as input.
- The string x will be a **suffix of the original** input that has not yet been parsed.
  (The original input is a suffix of itself.)
- The job of the parse function is to determine whether a **prefix** of x can be generated by the corresponding non-terminal.

Parsing Function Picture



Parser Function Hunger

- There may be more than one y that works. Our convention is that the parse function will always **prefer the longest** such y.



Parse Function Jobs

- Here x is any string.
- (parse-S x) determines whether a *prefix* of x can be generated from S.
- (parse-L x) determines whether a *prefix* of x can be generated from L.
- These functions will be mutually recursive.

1	S → (L)
2	L → ε
3	L → S L

parse-S

Anything S generates must start with the only production with S on the left:

$S \rightarrow (L)$

If S ultimately generates a prefix of string x, then x **must** start with **(**. So (parse-S x) will check that first.

Note that x could be empty, in which case (parse-S x) **fails**.

But if x begins with **(**, say x = **(** y for some y,

then parse-S calls (parse-L y) …

parse-S calls parse-L

- parse-S seeing input **(** y calls (parse-L y).
- (parse-L y) could do one of the following:

fail: y can't be generated from L

succeed: Some *prefix* u of y can be generated from L, with *residue* v,
i.e. y = u v and u is generated by L.

If parse-L succeeds, it is back to parse-S, to determine whether z begins with a matching **)**.

If so, (parse-S x) **succeeds**, leaving the residue for its caller.

If there is no matching **)**, (parse-S x) **fails** with residue x.

parse-L

- parseL has to worry about two rules:
  L → S L and L → ε.

- Remember that parse functions are “hungry”: they want to “eat” as much input as possible.

- So parse-L begins by immediately calling parse-S.
  - If that succeeds, then parse-L calls itself recursively.
  - If not, parse-L succeeds without eating any input, corresponding to the second rule.
  - Thus parse-L **always succeeds**, one way or the other, whereas parse-S may fail.

Implementation Details

Each parse function must return two things:

1. An indication of whether it succeeded or failed.
2. The residual unparsed input.

As we plan to code these functions using racket, we’ll create a **data abstraction** called **Outcome.**

Outcome data abstraction

- For now, an **Outcome** consists of only two things:

    success or failure

    residue of input

In future applications, we could add more things to an Outcome, which is one of the reasons we want a data abstraction and not a plain old list.

Constructors for an Outcome

internal

```
(define (make-outcome result residue)
    (list result residue))
```

for general use

```
(define (succeed residue)
    (make-outcome 'success residue))

(define (fail residue)
    (make-outcome 'failure residue))
```

Accessors for an Outcome

internal

```
(define (get-result Outcome)
        (first Outcome))

(define (get-residue Outcome)
        (second Outcome))
```

for general use

```
(define (failed? Outcome)
        (equal? 'failure (get-result Outcome)))

(define (succeeded? Outcome)
        (equal? 'success (get-result Outcome)))
```

Two other housekeeping functions

```
(define (left-paren? char)   (char=? #\( char))

(define (right-paren? char) (char=? #\) char))
```

These prevent the code from being cluttered with bare “magic” characters.

; Parse function for rule S -> (L)

```
(define (parse-S input)
  (cond
    [(null? input) (fail input)]
    [(left-paren? (first input))
     (let (
          (L1 (parse-L (rest input))) ;; can't fail
          )
       (cond
         [(null? (get-residue L1)) (fail input)]
         [(right-paren? (first (get-residue L1)))
                (succeed (rest (get-residue L1)))]
         [else (fail input)])

       )]
    [else (fail input)]))
```

; Parse function for rules L -> S L and L -> empty

```
(define (parse-L input)
  (let (
        (S1 (parse-S input))
        )
    (if (succeeded? S1)
        (let (
              (L2 (parse-L (get-residue S1))) ;; can't fail
              )
          (succeed (get-residue L2))
          )
        (succeed input))))

;; parse-L can't fail
```

; Parse function for rules L -> S L and L -> empty

```
(define (parse input-string)
  (let* (
        (outcome (parse-S (string->list input-string)))
        (residue (get-residue outcome))
        )
    (cond
      [(and (succeeded? outcome) (null? residue))
              "fully successful"]
      [(succeeded? outcome)
              (string-append "successful, with residue: "
                             (list->string residue))]
      [else "unsuccessful"])))
```

Some Unit Tests

```
(check-expect (parse "()")            "fully successful")
(check-expect (parse "(()())")        "fully successful")
(check-expect (parse "(()()())")      "fully successful")
(check-expect (parse "((())())")      "fully successful")
(check-expect (parse "((())(()))")    "fully successful")
(check-expect (parse "((()())(()))")  "fully successful")
(check-expect (parse "(()()))")       "successful, with residue: )")
(check-expect (parse "()()")          "successful, with residue: ()")
(check-expect (parse "(())()")        "successful, with residue: ()")
(check-expect (parse "(")             "unsuccessful")
(check-expect (parse  ")")            "unsuccessful")
(check-expect (parse  ")(")           "unsuccessful")
```