Assignment 3: Predicate Logic

Due: 11:00am, Wednesday, September 26

- Emails about this assignment should be directed to `cs81help@cs.hmc.edu`.
- The usual collaboration rules apply. You may *discuss* an exercise with any other student(s) currently taking CS 81 as long as:
  - You contribute equally;
  - You come away from this discussion only with *understanding in your head* — no written materials or computer notes may be retained;
  - Your submission is authored solely by you, on a separate occasion.
- You should refer only to materials from this semester of CS 81 (lecture notes, handouts, textbooks, grutors, profs, etc.).
- Bring a writeup/printout to class on the due date. Illegible answers will get no credit. (For this reason, the grutors like you to use LaTeX. But it's up to you.)
- Make sure your submission includes your name!

Reading

Review Sections 2.1 Huth & Ryan. Also read the sample proofs in Section 2.3 and make sure you understand how they fit together!

1 Thinking in Predicate Logic

Do Exercise 2.1.4(a-e,i), found on page 158 of Huth and Ryan. Note that some relations like "spouse" don't have a predicate symbol for you to use; you'll have to encode these as logical combinations involving the predicates you do have.

2 Formal Logic

Give natural deduction proofs of the following:

1. $\exists x.\,(R(x) \to B(x)) \vdash (\forall x.\,R(x)) \to (\exists x.\,B(x))$

   (e.g., $R(x)$ = "x is red", $B(x)$ = "x is bacon-flavored")

2. $(\forall x.\,L(x)) \vee (\forall x.F(x)) \vdash \forall x.\,(L(x) \vee F(x))$

   (e.g., $L(x)$ = "x is lost", $F(x)$ = "x is found")

3. $\forall x.\,(I(x) \vee U(x)) \vdash (\forall x.\,I(x)) \vee (\exists x.U(x))$

   (e.g., $I(x)$ = "x is interesting", $U(x)$ = "x is unremarkable")

4. $\forall x.\,(p \to Q(x)) \vdash p \to \forall x.Q(x)$

   (e.g., p = "I'm wearing earplugs", $Q(x)$ = "x is quiet")

5. $\vdash \exists x.\,(D(x) \to \forall y.\,D(y))$

   (The "drinker's paradox": there is at least one person satisfying "if he/she drinks beer, then everyone drinks beer.")

6. $\neg(\exists x.\,U(x)) \vee (\forall x.\,V(x)),\ p \to \forall x.\,D(x)\ \vdash\ \forall y.\forall z.\,((\neg U(z) \vee V(y)) \wedge (p \to D(y)))$

   (e.g., $U(x)$ = "x is a unicorn", $V(x)$ = "x is a vegetable", p = "I'm hungry", $D(x)$ = "x is delicious")

7. $\forall x.\,\neg S(x,x),\ \forall x.\forall y.\forall z.\,S(x,y) \wedge S(y,z) \to S(x,z)\ \vdash\ \forall x.\forall y.S(x,y) \to \neg S(y,x)$

   (e.g., $S(x,y)$ = "movie x is a sequel to movie y")

Extra Credit [10%]

The formula $\neg\neg p \to p$ is a tautology, but it is not provable in constructive (a.k.a. intuitionistic) logic. Any proof involves using a "classical" rule like proof-by-contradiction, or the law of the excluded middle, or (for the shortest proof) $\neg\neg$-elimination.

But even constructive logicians will agree that the formula "isn't false," because

$$\neg\neg(\neg\neg p \to p)$$

is provable constructively, without any non-constructive rules. Provide a constructive proof. (You will need $\neg$-introduction, but that's not the same as proof-by-contradiction.)

[This problem is nontrivial; don't spend too much time on it unless you find it interesting!]