Assignment 6: Temporal Logic and Cardinality

Due: 11:00am, Wednesday, October 31

- Emails about this assignment should be directed to `cs81help@cs.hmc.edu`.
- The usual collaboration rules apply. You may *discuss* an exercise with any other student(s) currently taking CS 81 as long as:
  - You contribute equally;
  - You come away from this discussion only with *understanding in your head* — no written materials or computer notes may be retained;
  - Your submission is authored solely by you, on a separate occasion.
- You should refer only to materials from this semester of CS 81 (lecture notes, handouts, textbooks, grutors, profs, etc.).
- Bring a writeup/printout to class on the due date. Illegible answers will get no credit.
- Make sure your submission includes your name!

1 Temporal Logic

(CTL was described in class. It also appears in Section 3.4 of Huth & Ryan.)

	Eventually True	Invariably True
Some Path	EF p Possibly	EG p Potentially Always
All Paths	AF p Inevitably	AG p Invariably

Consider the possible execution paths in the following piece of C/C++/Java-style code:

```
int a[100] = { 0, 0, 0, ..., 0 };  // Array is initially all zeros

while (true) {
    int k = user_input();   // Assume this user input will be a number
                            //      between 0 and 99 inclusive!

    if (a[k] == 1) {
         // Reset the array back to zeros
         for (int i = 0; i < 100; ++i)
              a[i] = 0;
    } else {
         // Set the k-th element of the array to 1
         a[k] = 1;
    }
}
```

Which of the following logical propositions are true (starting at the beginning of the program)? Explain your answer; you do not need to provide a full formal proof, but your explanation should be clear and convincing.

1. $\mathcal{A}\mathsf{G}\,(\mathsf{a}[42] = 0)$
2. $\mathcal{A}\mathsf{G}\left(\sum_{j=0}^{j<100} \mathsf{a}[j] \quad < \quad 100\right)$
3. $\mathsf{EF}\,(\mathsf{a}[42] = 1)$
4. $\mathcal{A}\mathsf{F}\,(\mathsf{a}[42] = 1)$
5. $\mathsf{EG}\,(\mathsf{a}[42] = 0)$
6. $\mathcal{A}\mathsf{G}\,(\mathcal{A}\mathsf{F}\,(\mathsf{a}[42] = 0))$
7. $\mathcal{A}\mathsf{G}\,(\mathsf{EF}\,(\mathsf{a}[42] = 1))$
8. $\mathsf{EF}\,(\mathcal{A}\mathsf{G}\,(\mathsf{a}[42] = 0))$

2 Cardinality

Carefully argue whether the following sets are countable or not.

1. The set of partial functions from $\mathbb{N}$ to $\mathbb{N}$ whose support is a finite set.

   [A total function is one that, for any input, produces an output. A *partial* function is one that, for some or all particular inputs, always returns “don’t know” or “undefined” (often written $\bot$ although it’s not a truth value) rather than returning an output. The *support* of a partial function (sometimes called the domain or domain of definition) is the collection of inputs that provide non-$\bot$ output.

   For example, we could have a partial function $\mathsf{sqrt} : \mathbb{R} \rightharpoonup \mathbb{R}$ defined by

   $$\mathsf{sqrt}(x) := \begin{cases} \sqrt{x} & \text{if } x \geqslant 0; \\ \bot & \text{otherwise.} \end{cases}$$

   The support of sqrt is then the set of all nonnegative real numbers.]

2. The set all valid Hoare Logic proofs.

3. The set of flow networks with rational capacities (directed graphs with finitely many nodes and edges, where each directed edge is labeled with a rational number called the “capacity” of that edge).

4. The set of Java programs that typecheck (i.e., that succesfully compile to a Java .class file)