# Assignment 7: Strings, Languages, and Induction

Due: 11:00am, Wednesday, November 7

- Emails about this assignment should be directed to `cs81help@cs.hmc.edu`.
- The usual collaboration rules apply. You may *discuss* an exercise with any other student(s) currently taking CS 81 as long as:
  - You contribute equally;
  - You come away from this discussion only with *understanding in your head* — no written materials or computer notes may be retained;
  - Your submission is authored solely by you, on a separate occasion.
- You should refer only to materials from this semester of CS 81 (lecture notes, handouts, textbooks, grutors, profs, etc.).
- Bring a writeup/printout to class on the due date. Illegible answers will get no credit.
- Make sure your submission includes your name!

## 1 Structural Induction on Programs

The midterm reviewed the imperative language of "commands" in our Hoare Logic proofs, and asked how induction could be used to prove that every command satisfies some property

Show that $\top$ works as a postcondition for every command, by using structural induction to prove that every command c satisfies:

"for every logical precondition $A$, there is
a Hoare Logic proof whose conclusion is $\{A\}\,c\,\{\top\}$"

Your proof should be in "mathematical English" (rather than a natural deduction or tableau proof). For reference, the relevant rules of Hoare Logic are repeated here:

$$\frac{}{\{P[e/x]\}\ \texttt{x = e}\ \{P\}}\ \text{ASSIGN} \qquad \frac{P \to P' \quad \{P'\}\,c\,\{Q'\} \quad Q' \to Q}{\{P\}\ c\ \{Q\}}\ \text{IMPLIED}$$

$$\frac{\{P\}\,c_1\,\{R\} \quad \{R\}\,c_2\,\{Q\}}{\{P\}\ c_1;c_2\ \{Q\}}\ \text{COMPOSITION} \qquad \frac{\{P \wedge e\}\,c_1\,\{Q\} \quad \{P \wedge \neg e\}\,c_2\,\{Q\}}{\{P\}\ \texttt{if}\ (e)\texttt{:}\ c_1\ \texttt{else:}\ c_2\ \{Q\}}\ \text{IF}$$

$$\frac{\{I \wedge e\}\ c\ \{I\}}{\{I\}\ \texttt{while}\ (e)\texttt{:}\ c\ \{I \wedge \neg e\}}\ \text{WHILE}$$

## 2 Deterministic State Machines

Use the idea of derivatives/intrinsic states to draw "minimal" deterministic state machines for the following languages:

- $L = \{ w \in \{a, b\}^* \mid w$ has an even number of a's and an odd number of b's $\}$
- $L = \{ a^n b^n \mid n \geqslant 0 \}$

## 3 Nondeterministic State Machines

Draw NFAs (NDFSMs) for the following languages, with the specified number of states. You may assume the alphabet is always $\{0, 1\}$.

1. The language $\{ w \mid w$ ends in 00 $\}$, 3 states
2. The language $\{ w \mid w$ contains the substring 0101 $\}$, five states
3. The language $\{ w \mid w$ contains an even number of 0s or exactly two 1s $\}$, six states
4. The language $\{0\}$, two states
5. The language $0^*1^*0^+$, three states
6. The language $1^*(001^+)^*$, three states
7. The language $\{\epsilon\}$, one state
8. The language $0^*$, one state