CS 81
Computability and Logic

September 5, 2012

Please fill out the survey
and then skim the syllabus
while I'm taking pictures...

CS 81
Computability and Logic

An Introduction to Formal Systems

Proofs vs. Computation

Defining the System

Well-Formed Formulas, Axioms/Inference Rules	Well-Formed Programs, Language Definition

Specific Instances

Assumptions Proof Step 1 Proof Step 2 ... Conclusion	Initial Conditions Computation Step 1 Computation Step 2 ... Result

Logic: A Selective History

Aristotle

Boole

Frege

Russell

Hilbert

Gödel

Church/Turing

Logic?

This dog is a father
This dog is mine

Therefore, this dog is my father

Logic?

Prof. Ran is Prof. Libeskind-Hadas
This is Prof. Libeskind-Hadas

Therefore, this is Prof. Ran

Man is animal
Fluffy is animal

Therefore, Fluffy is Man

Logic?

Steelworkers are often unionized

They rarely have missing electrons

Language is Ambiguous

John saw a picture
of the prettiest girl he had ever seen
hanging on a locker door.

Logic: A Selective History

Aristotle

Boole

Frege

Russell

Hilbert

Gödel

Church/Turing

Logic is Subtle

Put $U := \{ S \mid S \text{ is a set} \}$

Logic: A Selective History

Aristotle

Boole

Frege

Russell

Hilbert

Gödel

Church/Turing

An Inductively Defined Set

I'm thinking of a set S.

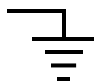
0 is in S.

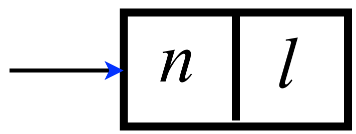
If n is in S, then n' is in S.

S is the smallest set obeying these rules.

(Equivalently: every element of S can be shown to belong to S by using the above rules finitely many times.)

An Inductively Defined Set

 is in L.

If (n is an integer) and (l is in L),
then 

is in L.

L is the smallest set obeying these rules.

(Equivalently: every element of L can be shown to belong to L by using the above rules finitely many times.)

An Inductively Defined Set

The empty string is in P.

If A and B are in P, then so is (A)B

P is the smallest set obeying these rules.

(Equivalently: every element of P can be shown to belong to P by using these rules finitely many times.)

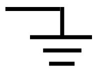
Structural Induction

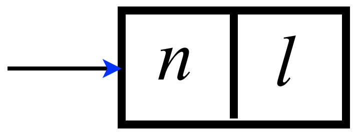
If the construction of an inductively defined set guarantees/preserves some property

then

every element of the set has that property.

Are there infinite lists in L?

 in L is finite? ✔

If n an integer and l in L. is finite
then 

is is finite? ✔

QED

Is Everything in P balanced?

The empty string in P is balanced? ✔

If A and B in P, are balanced
then (A)B in P is balanced? ✔

QED

An Inductively Defined Set

k "obeys the sum" formula: $$\sum_{i=0}^{k} i = \frac{k(k+1)}{2}$$

0 in N. obeys the sum? ✔

If n in N, obeys the sum
then n' in N. obeys the sum ✔

So every element of N obeys the sum formula. QED

2D Points

An inductive definition for a set S:

1. $(3, 5) \in S$.
2. If $(x, y) \in S$ then $(x + 2, y) \in S$.
3. If $(x, y) \in S$ then $(-x, y) \in S$.
4. If $(x, y) \in S$ then $(y, x) \in S$.

Is (-3,3) in S?

Is (168,-27) in S?

Can we prove it?

Propositional Logic

Proposition / Formula

An statement that could
be argued as true or false

Do You Recognize These Symbols?

$$(\text{CS 60} \lor \text{CS 42} \lor \text{CS 52})$$

$$\land$$

$$(\neg\text{Math 55} \to \text{CS 55})$$

$$\land$$

$$\neg\text{CS 81.}$$

Well-Formed Formulas

(an inductive definition!)

p, q, r, etc., are WFFs.

$\top$ and $\bot$ are WFFs.

If A is a WFF then so is $\neg$A.

If A and B are WFFs then so is (A$\wedge$B).

If A and B are WFFs then so is (A$\vee$B).

If A and B are WFFs then so is (A$\rightarrow$B).

Exercise

Choose specific propositions for p and q, and express these formulas in English.

$(p \wedge q) \rightarrow (q \wedge p)$

$p \rightarrow p$

$p \vee \neg p$

$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$

$\neg\neg(p \vee \neg p)$

Truth vs. Provability

A is true:

$$\models A$$

A is provable:

$$\vdash A$$

How should these relate?

Which is easier to demonstrate?

Theorems

(An inductively defined Set!)

- Axioms/Axiom Schemes:

  ... is a theorem.

  ... is a theorem.

- Rules of Inference:

  If ... is a theorem and ... is a theorem
  then ... is also a theorem.

Example:

Axiom Schemes (For any <u>formulas</u> F, G, and H):

$$\vdash F \rightarrow (G \rightarrow F)$$

$$\vdash (F \rightarrow G \rightarrow H) \rightarrow (F \rightarrow G) \rightarrow (F \rightarrow H)$$

$$\vdash (\neg G \rightarrow \neg F) \rightarrow (F \rightarrow G)$$

Rule of Inference: (For any formulas F and G):

If $\vdash F$ and $\vdash F \rightarrow G$ then $\vdash G$.

Theorems

(An inductively defined Set!)

Axioms/Axiom Schemes:

... is a theorem.

... is a theorem.

} true?

Rules of Inference:

If ... is a theorem and ... is a theorem
then ... is also a theorem.

} truth-preserving?