

Worksheet: Decidability Dialogue

Q: Can you remind me what a “decision problem” is?

A:

Q: Why does Prof. Stone keep jumping back and forth between “decision problems” and “formal languages”? How can a set of finite strings be a problem?

A:

Q: Thanks! That’s helpful. Could you remind me what it officially means for a decision problem (or formal language) to be *decidable*? (I think the definition involves Turing Machines somehow...)

A:

Q: I understand that the Church-Turing Thesis says that if a problem is intuitively solvable, then there is a Turing Machine that solves it. Does it necessarily follow that if a problem is solvable, there is a Java program that solves it? If so, is there a definition of decidability that doesn’t involve TMs?

A:

Q: So if problem is “undecidable,” that means...

A:

Q: Bertrand Russell proved that not every property defined a set, because not-a-member-of-yourself is a property of sets, but the set of “all sets that are not members of themselves” had a problem. Could you remind me what that problem was?

A:

Q: That’s right. It seems similar to our proof that there’s no algorithm to decide whether an arbitrary TM accepts an arbitrary string. If such an algorithm $\text{DecideAcceptance}(M,w)$ existed, we could use it to define a boolean function $\text{Cant}(M)$ that such that Cant says yes to all machines that say no to their own program. Why did this lead to a contradiction?

A:

Q: A similar argument shows that there’s no algorithm to decide whether a TM *halts* on a given string. But these Cant-style proofs are kind of weird. We argued that there’s an easier way to show that other problems are unsolvable, To show that a decision problem P is undecidable, prove that if we had a decision algorithm (or TM) that could answer question P , then ...

A:

Q: What is the most common method of showing this?

A:

Q: What does it mean for a problem to be *semidecidable*?

A:

Q: What is the relationship between semidecidable, undecidable, and decidable?

A:

Q: Suppose I want to show that a problem P is not semidecidable. Is it enough to prove that “if P were semidecidable, then TM-acceptance is semidecidable”?

A:

Q: Well, then what do I need to do?

A: