Worksheet: Tableau Proofs

1. Suppose you are trying to prove some formula C. What is something you could "safely" do right away (i.e., might not help, but definitely won't hurt), if you have the assumption

   - $A \wedge B$?

   - $A \vee B$?

2. We say that a formula is a *stacking* formula if it is equivalent to a conjunction, and a *splitting* formula if it is equivalent to a disjunction.[1] Which of the following formulas are splitting and which are stacking?

   - $A \to B$

   - $\neg(A \wedge B)$

   - $\neg(A \vee B)$

   - $\neg(A \to B)$

   - $\neg\neg A$. (This is neither. But how could we simplify it?)

[1] Equivalent in a nontrivial way, of course, since any formula A satisfies $A \equiv A \wedge \top \equiv A \vee \bot$.

3. A (classical logic) *tableau proof* is arranged as follows:

   - We start with the premises and the *negation* of the conclusion, and try to prove $\perp$. So, **it's always a proof by contradiction.**
   - We build a tree (creating and extending paths) by splitting or stacking; once a formula is split or stacked we can "check it off" and ignore it thereafter.
   - A path from the root is *closed* if it contains a contradiction (i.e., both r and $\neg r$); we stop extending this path.
   - The process stops with success if all paths are closed. (If we get stuck before all paths close, we can use the open path(s) to derive a counterexample.)

   Give tableau proofs for:

   - $p \to q \to p$

   - $(p \vee q) \wedge \neg q \to p$

- $(p \to q) \to (\neg q \to \neg p)$

- $((p \to q) \to p) \to p$

- $((p \to q) \wedge (\neg p \to q)) \to q$

- $((p \to q) \to r) \to (p \to (q \to r))$

- $((w \to m) \land (x \to a)) \to ((x \to m) \lor (w \to a))$

4. What splitting/stacking/simplifying can we do in a tableau proof with:
   - $\exists x.\ P(x)$
   - $\forall x.\ P(x)$
   - $\neg\exists x.\ P(x)$
   - $\neg\forall x.\ P(x)$

5. Give a tableau proof for:

    - $\forall x.\, P(x) \vdash \neg \exists x.\, \neg P(x)$.

    - $\exists z.\, \forall w.\, Q(z, w) \vdash \forall y.\, \exists x.\, Q(x, y)$