1. [35 Points] Headbook! Headbook, the wildly successful Shmorbodian social networking site, models its social networks as unweighted undirected graphs, where vertices are people and edges are friendships between people. Headbook computes a number of measurements on its social networks and one important one is called the diameter. The diameter of a graph is the maximum distance between all pairs of vertices. Recall that the distance between a pair of vertices is the length of a shortest path between them. So the diameter is the maximum of all the pairwise distances between points, where each distance is the length of a shortest path between those points. For this reason, distance is called a “maxi-min” measure because its the maximum of minimums (shortest paths). A small diameter indicates that the network is a tight cluster of friends whereas a large diameter suggests a more “spread out” collection of friends. Headbook represents its graphs using adjacency lists.

(a) Describe an algorithm for computing the diameter of an undirected graph and derive its running time. Your algorithm should be simple and run in time $O(n^k)$ for some constant $k$.

(b) It turns out that some social networks have no cycles (they are acyclic). Describe an algorithm for computing the diameter of an undirected acyclic graph. Your algorithm should run in time $O(n)$ and you should explain why your algorithm runs in this much time. (Hint: There are several different and equally good solutions to this problem. One is a modification of the key idea in the topological sorting algorithm. Another is to use depth-first or breadth-first search some small number of times.)

(c) In a few sentences, give a convincing explanation of the correctness of your algorithm. (That is, we’re asking for a “sketch” of how the proof would go.)