

Algorithms
Computer Science 140 & Mathematics 168
Spring 2012
Homework 6
Due Tuesday, February 28

1. **[35 Points] Headbook!** Headbook, the wildly successful Shmorbodian social networking site, models its social networks as unweighted undirected graphs, where vertices are people and edges are friendships between people. Headbook computes a number of measurements on its social networks and one important one is called the *diameter*. The *diameter* of a graph is the maximum distance between all pairs of vertices. Recall that the distance between a pair of vertices is the length of a *shortest path* between them. So the diameter is the maximum of all the pairwise distances between points, where each distance is the length of a shortest path between those points. For this reason, distance is called a “maxi-min” measure because its the maximum of minimums (shortest paths). A small diameter indicates that the network is a tight cluster of friends whereas a large diameter suggests a more “spread out” collection of friends. Headbook represents its graphs using adjacency lists.
 - (a) Describe an algorithm for computing the diameter of an undirected graph and derive its running time. Your algorithm should be simple and run in time $O(n^k)$ for some constant k .
 - (b) It turns out that some social networks have no cycles (they are *acyclic*). Describe an algorithm for computing the diameter of an undirected acyclic graph. Your algorithm should run in time $O(n)$ and you should explain why your algorithm runs in this much time. (Hint: There are several different and equally good solutions to this problem. One is a modification of the key idea in the topological sorting algorithm. Another is to use depth-first or breadth-first search some small number of times.)
 - (c) In a few sentences, give a convincing explanation of the correctness of your algorithm. (That is, we’re asking for a “sketch” of how the proof would go.)