Puzzling and Prolog

February 13–14, 2012
CS 60: Principles of Computer Science

Assignment 3 due: Prolog Intro
Assignment 4 due Monday February 20: Prolog Puzzles
Also on Monday, February 20: Guest Lecture by Paul Ruvolo (HMC '03)

Main Strategy So Far

Give constraints that *describe* solution(s)

```
aunt(A,N) :- parent(P,N), sib(A,P), female(A).

length(L,N) :- L = [], N = 0.
length(L,N) :- L = [F|R], length(R,M), N is M+1.
```

Prolog *does* all the work to find or check solutions.

Examples: pos and inrange

```
% N is positive integer if ...

pos(1).
pos(N) :- pos(M), N is M+1.

% K is in the valid range  Lo...Hi  inclusive if
%   K is Lo, or else if K is in (Lo+1)...K inclusive.

inrange(Lo,Hi,K) :- Lo =< Hi, K = Lo.
inrange(Lo,Hi,K) :- Lo =< Hi, Next is Lo+1, inrange(Next,Hi,K).
```

Name: "Quiz"

1. `insertOne(X, L, M)` — true when `M` results from putting `X` in list `L`. (Use `removeOne`.)

```
insertOne(X, L, M) :-
insertOne(X, L, M) :- removeOne(X, M, L).
```

2. `perm(L,M)` — true when `M` is a permutation (shuffling) of `L`. (Use `insertOne`)

```
perm([], []).
perm([F|R], M) :-
perm([F|R], M) :- perm(R,P), insertOne(F,P,M)
```

3. `sorted(L)` — true when `L` is a list in sorted (`=<`) order.

```
sorted([]).
sorted(        ) % Another base case
sorted( [_] ) % Another base case
sorted([X,Y|Rest]) :-
sorted([X,Y|Rest]) :- X =< Y, sorted([Y|Rest]).
```

The Zebra Puzzle

There are five houses:

- ✓ The nationalities are norwegian, brit, swede, dane, german
- ✓ The pets are dog, bird, zebra, cat, horse
- ✓ The cigars are pallmall, winfield, dunhill, rothmans, marlboro
- ✓ The beverages are tea, coffee, milk, water, beer
- ✓ The house colors are red, green, yellow, blue, white

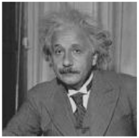
We know that

1. The Norwegian lives in the first house. ...

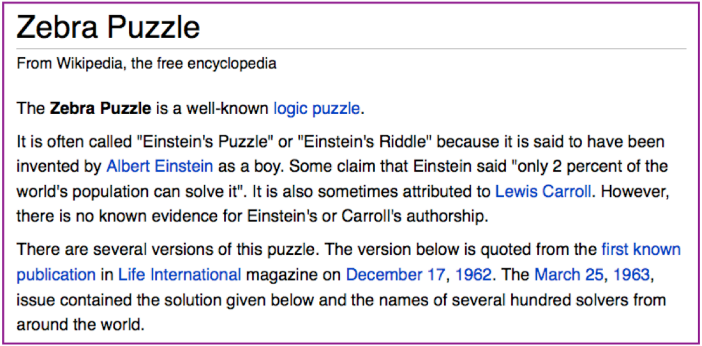
Who owns the zebra?

Puzzle History

Often attributed to Einstein



But...



Zebra Puzzle

From Wikipedia, the free encyclopedia

The **Zebra Puzzle** is a well-known logic puzzle.

It is often called "Einstein's Puzzle" or "Einstein's Riddle" because it is said to have been invented by Albert Einstein as a boy. Some claim that Einstein said "only 2 percent of the world's population can solve it". It is also sometimes attributed to Lewis Carroll. However, there is no known evidence for Einstein's or Carroll's authorship.

There are several versions of this puzzle. The version below is quoted from the first known publication in Life International magazine on December 17, 1962. The March 25, 1963, issue contained the solution given below and the names of several hundred solvers from around the world.

The Boring Constraints

```
zebra(H) :-
    ...
    houses(H),
    ... .
```

```
houses( [H1, H2, H3, H4, H5] ) :-
     H1 = [ N1, P1, S1, B1, C1 ],
     H2 = [ N2, P2, S2, B2, C2 ],
     H3 = [ N3, P3, S3, B3, C3 ],
     H4 = [ N4, P4, S4, B4, C4 ],
     H5 = [ N5, P5, S5, B5, C5 ],
     perm( [N1,N2,N3,N4,N5], [norwegian, brit, swede, dane, german] ),
     perm( [P1,P2,P3,P4,P5], [dog, bird, zebra, cat, horse] ),
     perm( [S1,S2,S3,S4,S5], [pallmall, winfield, dunhill, rothmans, marlboro] ),
     perm( [B1,B2,B3,B4,B5], [tea, coffee, milk, water, beer] ),
     perm( [C1,C2,C3,C4,C5], [red, green, yellow, blue, white] ),
```

Representing Constraints

What constraints does this express?

```
H = [ [norwegian, _, _, _, _],
      _,
      [_, _, _, milk, _],
      _,
      _]
```

7) How can we say "The owner of the green house drinks coffee"
   (Hint: use `member` and `H`)

```
member([_, _, _, coffee, green], H).
```

More Constraints

```
% left-to-right adjacency:
lr(L, R, [L, R | _]).
lr(L, R, [_ | Rest]) :- lr(L, R, Rest).

% unordered adjacency
nextTo(X, Y, List) :- lr(X, Y, List).
nextTo(X, Y, List) :- lr(Y, X, List).
```

6) "The Green house is next to, and left of, the White house."

```
nextTo([_,_,_,_,green], [_,_,_,_,white], H)
```

15a) "The Norwegian doesn't live by the red house." (Where should this constraint go?)

```
\+ nextTo([norwegian,_,_,_,_], [_,_,_,_,red], H)
```

Producing Nice Output

```
solve :-
  zebra( [ H1, H2, H3, H4, H5 ] ),
  write( ' first house: '), write(H1), nl,
  write( 'second house: '), write(H2), nl,
  write( ' third house: '), write(H3), nl,
  write( 'fourth house: '), write(H4), nl,
  write( ' fifth house: '), write(H5), nl.
```

Homework: The Mystery of the Spamwarts Express

Five people are traveling home in adjacent train seats for summer break
Each is from a different Claremont College (pomona, pitzer, hmc, cmc, scripps).
Each has a different name (algird, bruno, collette, dino, edwina).
Each has brought a different snack (jots, chocolate, donuts, pez, spam).

1. Dino and Bruno sat in the end seats.
2. Algird sat next to the student from HMC.
3. Collette sat next to friends with chocolate and donuts.
4. The HMC student brought spam as a snack and sat in the middle seat.
5. Chocolate was immediately to the left of pez.
6. Bruno, Dino, and Algird do not go to Scripps.
7. The Pomona student sat between the one with jots and the one with spam.
8. Dino did not sit next to the person with donuts.
9. The CMC student did not sit next to Edwina.

Prolog Shortcomings

```
% True when K evenly divides N

divides(N, K) :-  0 =:= N mod K.
```

Which query or queries will work properly?

1. `divides(42, 3).` ✔
2. `divides(42, 4).` ✔
3. `divides(42, K).` ✗
4. `divides(N, 42).` ✗

Strategy: Generate-and-Test

Problem: Our constraints involve arithmetic, negations, and (in)equalities that only work with "known" values.

Solution:

1. Have Prolog generate *all* potential values.
2. Check each to see if it works.

```
% Better
divides(N, K) :- inrange(1, N, K), 0 =:= N mod K.
```

```
% Best
divides(N, K) :- pos(N), inrange(1, N, K), 0 =:= N mod K.
```

Recall: Broken Definition of `oldest`

```
notoldest(X) :- age(X,AX), age(Y,AY), AX < AY.
oldest(X) :- \+ notoldest(X).
```

```
?- notoldest(X).
X = helga ;
X = helga ;
X = olf ;
X = uggette ;
X = uggette ;
X = uggette ;
X = ug ;
X = ug ;
X = ug ;
X = ug ;
X = matilda ;
X = matilda ;
...
```

```
?- oldest(helga).
false.

?- oldest(maggie).
false.

?- oldest(skugerina).
true.

?- oldest(X).
false.
```

Generate-and-Test to the Rescue!

```
notoldest(X) :- age(X,AX), age(Y,AY), AX < AY.
oldest(X) :- person(X), \+ notoldest(X).
```

```
?- notoldest(X).
X = helga ;
X = helga ;
X = olf ;
X = uggette ;
X = uggette ;
X = uggette ;
X = ug ;
X = ug ;
X = ug ;
X = ug ;
X = matilda ;
X = matilda ;
...
```

```
?- oldest(helga).
false.

?- oldest(maggie).
false.

?- oldest(skugerina).
true.

?- oldest(X).
X = skugerina ;
false.
```

Other Generate-and-Test Applications

```
%% sort(L,LSort)  --- True when LSort is a sorting of L

sort(L,LSort) :- perm(L, LSort), sorted(LSort).



%% subset(X,Y) --- True when Y contains the elements of
%%                 X, plus possibly others, in any order

subset(X,Y) :- perm(Y, NiceY), append(X, _, NiceY).
```

A Classic Logic Puzzle

Assign different digits to each letter, making the sum correct. (`M` is not 0.)

```
    S E N D
  + M O R E
  ---------
  M O N E Y
```

Solution

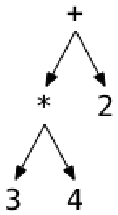
```
uniquedigits(L) :- subset(L, [0,1,2,3,4,5,6,7,8,9]).

add(A, B, C, Sum, Carry) :-
  Sum is (A + B + C) mod 10,
  Carry is (A + B + C) // 10.

money(Answer) :-
  % Generate:
  Answer = [S,E,N,D,M,O,R,Y],
  uniquedigits(Answer),

  % Test:
  M \== 0,
  add(0,  D, E, Y, C1),
  add(C1, N, R, E, C2),
  add(C2, E, O, N, C3),
  add(C3, S, M, O, C4),
  add(C4, 0, 0, M, 0).
```

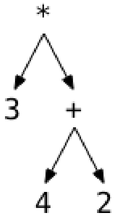
```
C4  C3  C2  C1
     S   E   N   D
+    M   O   R   E
-----------------
 M   O   N   E   Y
```

Arithmetic Expressions as Trees

Any arithmetic expression can be represented as a binary tree:

- ✓ Leaves are numbers
- ✓ Interior nodes are operators (+, -, *, or /)





`[+, [*, 3, 4], 2]` `[*, 3, [+, 4, 2]]`

Note: Here leaves are just numbers, rather than lists with empty subtrees.

Homework: The (Generalized) 24 Problem

Given:

1. A list of possible operators (usually `[+, -, *, /]`)
2. A list of integers (e.g., `[3, 4, 2]`)
3. A goal value

is there some arithmetic tree such that

- ✓ The leaves are the given integers (exactly once each)
- ✓ The interior nodes are from our possible operators
- ✓ The "value" of the tree is our goal value

```
?- solve([+,*,/], [2,3,4,6], 24, T).
T = [+,2,[+,4,[*,3,6]]] ;
T = [+,2,[+,4,[*,6,3]]] ;
...
T = [+,[/,2,3],[*,4,6]] ;
...
T = [+,[*,3,[+,4,2]],6] ;
...
```

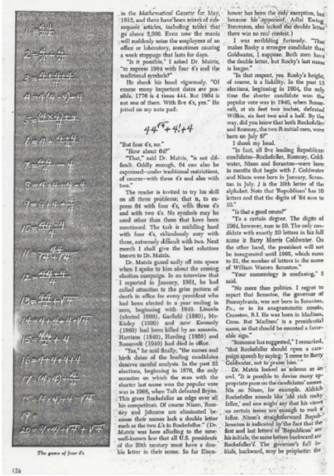
Twenty-Four as Generate-and-Test

- ✓ Partition the integers into two groups
- ✓ Recursively make each group into a tree
- ✓ Combine the two subtrees into a single tree

- ✓ Check whether it has the right goal value.

```
% eval(T,N) --- True when tree T evaluates to number N.

eval(R, R) :- number(R).
eval(['+', A, B], R) :- eval(A, AR), eval(B, BR), R is AR + BR.
eval(['*', A, B], R) :- eval(A, AR), eval(B, BR), R is AR * BR.
eval(['-', A, B], R) :- eval(A, AR), eval(B, BR), R is AR - BR.
eval(['/', A, B], R) :- eval(A, AR), eval(B, BR), BR\==0, R is AR // BR.
```

Related Puzzle: Four 4's



$1 = \frac{44}{44}$

$2 = \frac{4}{4} + \frac{4}{4}$

$3 = \frac{4+4+4}{4}$

$4 = 4(4-4)+4$

$5 = \frac{(4 \times 4)+4}{4}$

$6 = 4 + \frac{4+4}{4}$

$7 = \frac{44}{4} - 4$

$8 = 4+4+4-4$

$9 = 4+4+\frac{4}{4}$

$10 = \frac{44-4}{4}$

$11 = \frac{44}{\sqrt{4}+\sqrt{4}}$

$12 = \frac{44+4}{4}$

$13 = \frac{44}{4} + \sqrt{4}$

$14 = 4+4+4+\sqrt{4}$

$15 = \frac{44}{4} + 4$

$16 = 4+4+4+4$

$17 = (4 \times 4) + \frac{4}{4}$

$18 = (4 \times 4)+4-\sqrt{4}$

$19 =$

$20 = (4 \times 4)+\sqrt{4}+\sqrt{4}$

The game of four 4's

in the *Mathematical Gazette* for May, 1912, and there have been scores of subsequent articles, including tables that go above 2,000. Even now the mania will suddenly seize the employees of an office or laboratory, sometimes causing a work stoppage that lasts for days.

"Is it possible," I asked Dr. Matrix, "to express 1964 with four 4's and the traditional symbols?"

He shook his head vigorously. "Of course many important dates *are* possible. 1776 is 4 times 444. But 1964 is not one of them. With five 4's, yes." He jotted on my note pad:

$$44^{\sqrt{4}} + 4! + 4$$

"But four 4's, no."

"How about 64?"

"That," said Dr. Matrix, "is not difficult. Oddly enough, 64 can also be expressed—under traditional restrictions, of course—with three 4's and also with two."

The reader is invited to try his skill on all three problems; that is, to express 64 with four 4's, with three 4's and with two 4's. No symbols may be used other than those that have been mentioned. The task is middling hard with four 4's, ridiculously easy with three, extremely difficult with two. Next month I shall give the best solutions known to Dr. Matrix.

Dr. Matrix gazed sadly off into space when I spoke to him about the coming election campaign. In an interview that I reported in January, 1961, he had called attention to the grim pattern of death in office for every president who had been elected in a year ending in zero, beginning with 1840. Lincoln (elected 1860), Garfield (1880), McKinley (1900) and now Kennedy (1960) had been killed by an assassin. Harrison (1840), Harding (1920) and Roosevelt (1940) had died in office.

"Yes," he said finally, "the names and birth dates of the leading candidates deserve careful analysis. In the past 22 elections, beginning in 1876, the only occasion on which the man with the shorter last name won the popular vote was in 1908, when Taft defeated Bryan. This gives Rockefeller an edge over all his competitors. Of course Nixon, Romney and Johnson are eliminated because their names lack a double letter such as the two *L*'s in Rockefeller." (Dr. Matrix was here alluding to the now-well-known law that all U.S. presidents of the 20th century must have a double letter in their name. So far Eisenhower has been the only exception, but because his opponent, Adlai Ewing Stevenson, also lacked the double letter there was no real contest.)

I was scribbling furiously. "That makes Rocky a stronger candidate than Goldwater, I suppose. Both men have the double letter, but Rocky's last name is longer."

"In that respect, yes. Rocky's height, of course, is a liability. In the past 15 elections, beginning in 1904, the only time the shorter candidate won the popular vote was in 1940, when Roosevelt, at six feet two inches, defeated Willkie, six feet two and a half. By the way, did you know that both Rockefeller and Romney, the two *R*-initial men, were born on July 8?"

I shook my head.

"In fact, all five leading Republican candidates—Rockefeller, Romney, Goldwater, Nixon and Scranton—were born in months that begin with *J*. Goldwater and Nixon were born in January, Scranton in July. *J* is the 10th letter of the alphabet. Note that 'Republican' has 10 letters and that the digits of '64 sum to 10."

"Is that a good omen?"

"To a certain degree. The digits of 1964, however, sum to 20. The only candidate with exactly 20 letters in his full name is Barry Morris Goldwater. On the other hand, the president will not be inaugurated until 1965, which sums to 21, the number of letters in the name of William Warren Scranton."

"Your numerology is confusing," I said.

"No more than politics. I regret to report that Scranton, the governor of Pennsylvania, was not born in Scranton, Pa., or in its anagrammatic cousin, Cranston, R.I. He was born in Madison, Conn. But 'Madison' is a presidential name, so that should be counted a favorable sign."

"Someone has suggested," I remarked, "that Rockefeller should open a campaign speech by saying: 'I come to Barry Goldwater, not to praise him.' "

Dr. Matrix looked as solemn as an owl. "It is possible to devise many appropriate puns on the candidates' names. Nix on Nixon, for example. Aldrich Rockefeller sounds like 'old rich rocky feller,' and one might say that his views on certain issues are enough to rock a feller. Nixon's straightforward Republicanism is indicated by the fact that the first and last letters of 'Republican' are his initials; the same letters backward are Rockefeller's. The governor's full initials, backward, may be prophetic: the

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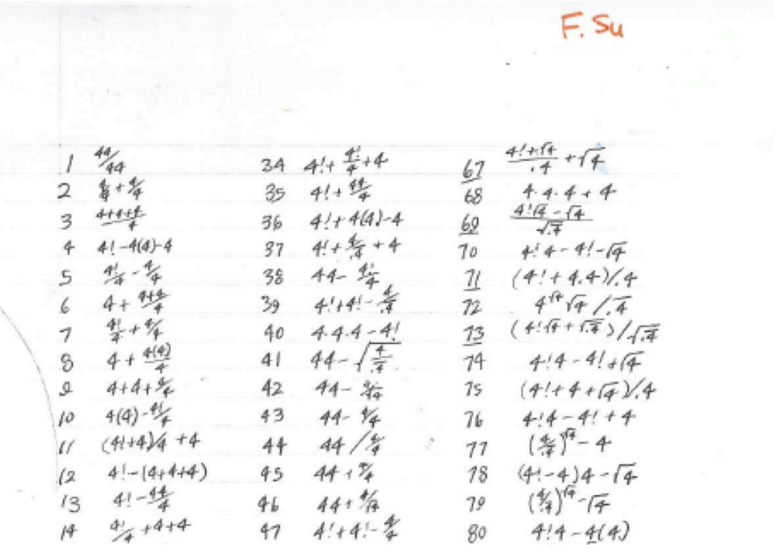
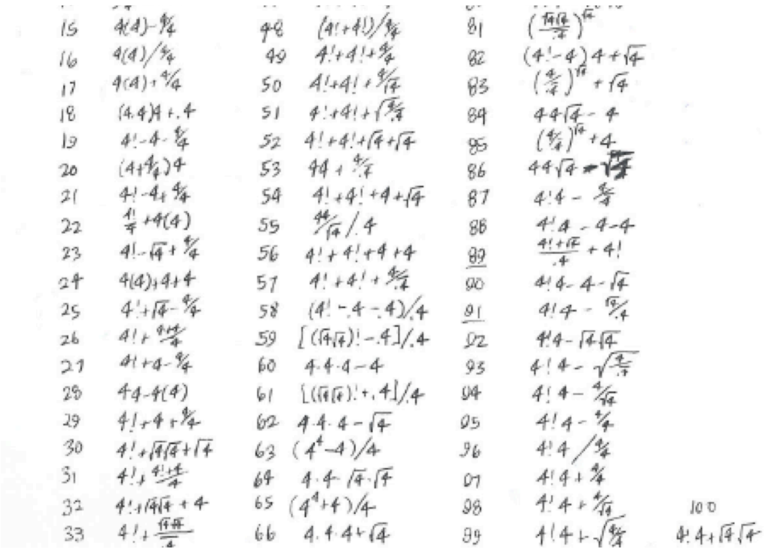
More Locally...



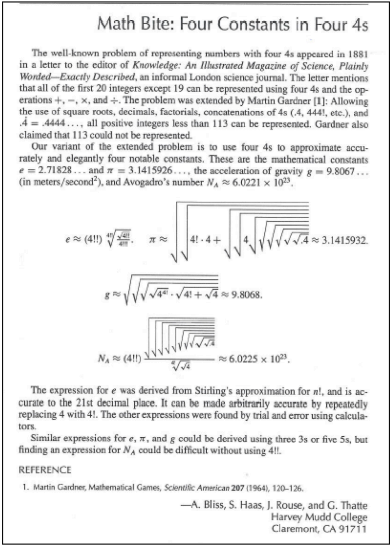
F. Su

1	$\frac{44}{44}$	34	$4! + \frac{4!}{4} + 4$	67	$\frac{4! + \sqrt{4}}{.4} + \sqrt{4}$
2	$\frac{4}{4} + \frac{4}{4}$	35	$4! + \frac{44}{4}$	68	$4 \cdot 4 \cdot 4 + 4$
3	$\frac{4+4+4}{4}$	36	$4! + 4(4) - 4$	69	$\frac{4!\sqrt{4} - \sqrt{4}}{\sqrt{.4}}$
4	$4! - 4(4) - 4$	37	$4! + \frac{4}{.4} + 4$	70	$4!4 - 4! - \sqrt{4}$
5	$\frac{4!}{4} - \frac{4}{4}$	38	$44 - \frac{4!}{4}$	71	$(4! + 4.4)/.4$
6	$4 + \frac{4+4}{4}$	39	$4! + 4! - \frac{4}{.4}$	72	$4^{\sqrt{4}}\sqrt{4}/.4$
7	$\frac{4!}{4} + \frac{4}{4}$	40	$4 \cdot 4 \cdot 4 - 4!$	73	$(4!\sqrt{4} + \sqrt{.4})/\sqrt{.4}$
8	$4 + \frac{4(4)}{4}$	41	$44 - \sqrt{\frac{4}{.4}}$	74	$4!4 - 4! + \sqrt{4}$
9	$4 + 4 + \frac{4}{4}$	42	$44 - \frac{4}{\sqrt{4}}$	75	$(4! + 4 + \sqrt{4})/.4$
10	$4(4) - \frac{4!}{4}$	43	$44 - \frac{4}{4}$	76	$4!4 - 4! + 4$
11	$(4! + 4)/4 + 4$	44	$44 / \frac{4}{4}$	77	$(\frac{4}{.4})^{\sqrt{4}} - 4$
12	$4! - (4 + 4 + 4)$	45	$44 + \frac{4}{4}$	78	$(4! - 4)4 - \sqrt{4}$
13	$4! - \frac{44}{4}$	46	$44 + \frac{4}{\sqrt{4}}$	79	$(\frac{4}{.4})^{\sqrt{4}} - \sqrt{4}$
14	$\frac{4!}{4} + 4 + 4$	47	$4! + 4! - \frac{4}{4}$	80	$4!4 - 4(4)$

More Locally…



A CS 60-inspired Paper (*Mathematics Magazine*, Oct. 2001)



Math Bite: Four Constants in Four 4s

The well-known problem of representing numbers with four 4s appeared in 1881 in a letter to the editor of *Knowledge: An Illustrated Magazine of Science, Plainly Worded—Exactly Described*, an informal London science journal. The letter mentions that all of the first 20 integers except 19 can be represented using four 4s and the operations +, −, ×, and ÷. The problem was extended by Martin Gardner [**1**]: Allowing the use of square roots, decimals, factorials, concatenations of 4s (.4, 444!, etc.), and $.\dot{4} = .4444\ldots$, all positive integers less than 113 can be represented. Gardner also claimed that 113 could not be represented.

Our variant of the extended problem is to use four 4s to approximate accurately and elegantly four notable constants. These are the mathematical constants $e = 2.71828\ldots$ and $\pi = 3.1415926\ldots$, the acceleration of gravity $g = 9.8067\ldots$ (in meters/second^2), and Avogadro's number $N_A \approx 6.0221 \times 10^{23}$.

$$e \approx (4!!)^{\sqrt[4!]{\frac{\sqrt{4!!}}{4!!!}}}, \quad \pi \approx \sqrt{\sqrt{4!\cdot 4 + \sqrt{\sqrt{4\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{.\dot{4}}}}}}}}}} \approx 3.1415932.$$

$$g \approx \sqrt{\sqrt{\sqrt{\sqrt{4^{4!}}\cdot\sqrt{4!}+\sqrt{4}}}} \approx 9.8068.$$

$$N_A \approx (4!!)\frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{4}}}}}}}}}}{\sqrt[4]{\sqrt{4}}} \approx 6.0225 \times 10^{23}.$$

The expression for e was derived from Stirling's approximation for $n!$, and is accurate to the 21st decimal place. It can be made arbitrarily accurate by repeatedly replacing 4 with 4!. The other expressions were found by trial and error using calculators.

Similar expressions for e, π, and g could be derived using three 3s or five 5s, but finding an expression for N_A could be difficult without using 4!!.

REFERENCE

1. Martin Gardner, Mathematical Games, *Scientific American* **207** (1964), 120–126.

—A. Bliss, S. Haas, J. Rouse, and G. Thatte
Harvey Mudd College
Claremont, CA 91711

One 4?

REPRESENTING NUMBERS USING ONLY ONE 4

DONALD E. KNUTH, California Institute of Technology

In the Mathematical Games section of *Scientific American*, January, 1964, Martin Gardner asks his readers to try their hand at the well-known problem of representing the number 64 using precisely four, three, and two 4's, respectively, combined with arithmetic operations. The "two-4" problem was first posed by J. A. Tierney in 1944 (see [1]), and some of the answers given were

$$64 = \sqrt{\sqrt{\sqrt{(4^{4!})}}} = \sqrt{(4^{\Gamma(4)})} = 4^{[\ln 4!]} = \phi(\phi(4^4)) = \phi(4!)^{\sqrt{4}} = 4^{\surd(\pi(4!))}.$$

It is the purpose of this note to show that the number 64 can be represented with only *one* 4, and indeed this may be true of all positive integers.

Using only the three operations $\surd$, [ ], and !, and the single digit 4, we have

$$64 = [\surd\surd\surd\surd\surd\surd\surd\surd\surd[\surd\surd\surd\surd\surd\surd\surd\surd\surd[\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd$$
$$[\surd\surd\surd\surd\surd\surd\surd\surd[\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd$$
$$[\surd[\surd[\surd\surd\surd\surd\surd(4!)!]!]!]!]!]!]!].$$

(In this representation, the notation $\sqrt{x}!$ stands for $\sqrt{(x!)}$, not $(\sqrt{x})!$.)

To prove this we will compute successively the quantities inside each of the nested brackets. The innermost brackets represent the number 5, since

$$5^{32} = \quad 232\ 83064\ 36538\ 69628\ 90625$$
$$24! = \ 6204\ 48401\ 73323\ 94393\ 60000$$
$$6^{32} = 79586\ 61109\ 94640\ 08843\ 91936$$

and hence $[\surd\surd\surd\surd\surd 24!] = 5$. The next level of brackets obviously gives 10, since $10 = [\sqrt{5!}]$. Proceeding similarly, we can evaluate the kth brackets to obtain the integer a_k, where

$$a_k^{2^t} < a_{k-1}! < (a_k + 1)^{2^t}$$

Zebra Puzzle as Generate-and-Test?

Why the difference?

Fast:

```
zebra(H) :-

  H = [[norwegian, _, _, _, _], _,
       [_, _, _, milk, _], _, _],
  member([brit, _, _, _, red], H),

  % ...lots of constraints...

  houses(H),
  \+ nextTo([norwegian,_,_,_,_], ...),
  \+ nextTo([norwegian,_,_,_,_], ...),
  \+ nextTo([norwegian,_,_,_,_], ...).
```

Slow:

```
zebra(H) :-
  houses(H),
  H = [[norwegian, _, _, _, _], _,
       [_, _, _, milk, _], _, _],
  member([brit, _, _, _, red], H),
  % ...lots of constraints...

  \+ nextTo([norwegian,_,_,_,_], ...),
  \+ nextTo([norwegian,_,_,_,_], ...),
  \+ nextTo([norwegian,_,_,_,_], ...).
```