Determine for each claim below if there is a mistake in the proof; if so, point it out explicitly. In the case where the claim itself is correct but the proof is wrong (incorrect or incomplete), provide a correct proof.

**Claim 1** *Every student taking CS81 will receive the same grade.*

*Proof:* We prove this by induction on the number \( N \) of students taking CS81.

The base case is \( N = 1 \) which is trivially true. Assume that the claim holds for \( N = k \), where \( k \geq 1 \). Consider the case where \( N = k + 1 \). All but the \((k + 1)\)-th student receive the same grade by the inductive hypothesis. Similarly, all but the first student receive the same grade. Thus, all \( k + 1 \) students receive the same grade. So, the claim for \( k + 1 \) holds.

This shows that all students taking CS81 receive the same grade. □

**Claim 2** *Two equals three.*

*Proof:* Consider the identity \( 2 = \frac{6}{5-2} \). By recursively applying the identity to 2 on the right-hand side, we obtain

\[
2 = \frac{6}{5-\frac{6}{5-\frac{6}{5-\ldots}}}.
\]  (1)

By the same token, consider the identity \( 3 = \frac{6}{5-3} \). By recursively applying the identity to 3 on the right-hand side, we obtain

\[
3 = \frac{6}{5-\frac{6}{5-\frac{6}{5-\ldots}}}.
\]  (2)

Since the right-hand sides of (1) and (2) are equal, this proves that 2 equals 3. □

**Claim 3** \( \sqrt{3} \) is irrational.

*Proof:* Suppose that \( \sqrt{3} \) is rational, that is, \( \sqrt{3} = \frac{a}{b} \), for two integers \( a \) and \( b \). Thus, \( a^2 = 3b^2 \). By arguments similar to the proof that \( \sqrt{2} \) is irrational, we arrive at a contradiction. This shows that \( \sqrt{3} \) is irrational. □

**Claim 4** *Any tree with at least two vertices has a vertex of degree one. Recall that a tree is a connected graph with no cycles.*

*Proof:* By induction on the number of vertices \( N \) of the tree.

The base case for \( N = 2 \) vertices is obvious. Suppose the claim holds for any tree with \( N = k \) vertices, where \( k \geq 2 \). Let \( T \) be a tree with \( N = k + 1 \) vertices. Pick a collection of \( k \) vertices from \( T \). By the inductive hypothesis, this collection contains a vertex of degree one.

This proves the claim. □