CS81 HW#1 Spring 2012
Out: 01/18/2012. Due: 01/25/2012 (before class)

Determine for each claim below if there is a mistake in the proof; if so, point it out explicitly. In the case where the claim itself is correct but the proof is wrong (incorrect or incomplete), provide a correct proof.

Claim 1 *Every student taking CS81 will receive the same grade.*

Proof: We prove this by induction on the number N of students taking CS81.

The base case is $N = 1$ which is trivially true. Assume that the claim holds for $N = k$, where $k \geq 1$. Consider the case where $N = k + 1$. All but the $(k + 1)$-th student receive the same grade by the inductive hypothesis. Similarly, all but the first student receive the same grade. Thus, all $k + 1$ students receive the same grade. So, the claim for $k + 1$ holds.

This shows that all students taking CS81 receive the same grade. □

Claim 2 *Two equals three.*

Proof: Consider the identity $2 = \frac{6}{5-2}$. By recursively applying the identity to 2 on the right-hand side, we obtain

$$2 = \frac{6}{5 - \frac{6}{5-\frac{6}{5-\dots}}}. \tag{1}$$

By the same token, consider the identity $3 = \frac{6}{5-3}$. By recursively applying the identity to 3 on the right-hand side, we obtain

$$3 = \frac{6}{5 - \frac{6}{5-\frac{6}{5-\dots}}}. \tag{2}$$

Since the right-hand sides of (1) and (2) are equal, this proves that 2 equals 3. □

Claim 3 $\sqrt{3}$ *is irrational.*

Proof: Suppose that $\sqrt{3}$ is rational, that is, $\sqrt{3} = a/b$, for two integers a and b. Thus, $a^2 = 3b^2$. By arguments similar to the proof that $\sqrt{2}$ is irrational, we arrive at a contradiction. This shows that $\sqrt{3}$ is irrational. □

Claim 4 *Any tree with at least two vertices has a vertex of degree one. Recall that a tree is a connected graph with no cycles.*

Proof: By induction on the number of vertices N of the tree.

The base case for $N = 2$ vertices is obvious. Suppose the claim holds for any tree with $N = k$ vertices, where $k \geq 2$. Let T be a tree with $N = k + 1$ vertices. Pick a collection of k vertices from T. By the inductive hypothesis, this collection contains a vertex of degree one.

This proves the claim. □