CS81 HW#2 Spring 2012 *Out: 01/25/2012. Due: 02/01/2012 (before class)*

Policy on homeworks

1. *Collaboration*: You may discuss a question with any other student currently taking CS81 provided: (i) both of you contribute equally; (ii) you come away from any discussion with an understanding in your mind (and no archived solution of any form is retained); (iii) your submission is your own work prepared by yourself on a separate occassion.
2. *Reference materials*: You should only refer to materials from this semester of CS81 (class notes, handouts, textbooks, grutors, instructor, etc).
3. *Submission*: Your submission should be legible or is prepared using TeX.

Prove or Disprove If **prove**, use only the Natural Deduction rules given below (augmented with the Copy rule). If **disprove**, provide a valuation or truth assignment which contradicts the assertion. Some of these problems are adapted from Huth and Ryan.

1. The other half of the Distributive Law $((a \wedge b) \vee (a \wedge c)) \vdash (a \wedge (b \vee c))$.
2. One-half of De Morgan's Law: $(\neg a \vee \neg b) \vdash \neg(a \wedge b)$.
3. Implication as Disjunction: $(a \rightarrow b) \vdash (\neg a \vee b)$.
4. Disjunction as Implication: $(\neg a \vee b) \vdash (a \rightarrow b)$.
5. Disjunction of Premises implies Disjunction of Goals:
$\{(a \rightarrow b), (c \rightarrow d)\} \vdash ((a \vee c) \rightarrow (b \vee d))$.
6. Conjunction of Premises implies Conjunction of Goals:
$\vdash (a \rightarrow b) \rightarrow ((c \rightarrow d) \rightarrow ((a \wedge c) \rightarrow (b \wedge d)))$.
7. Peirce's Law: $\vdash (((a \rightarrow b) \rightarrow a) \rightarrow a)$.
8. Mangled Form of Peirce's Law: $\vdash (b \rightarrow (a \rightarrow (b \rightarrow a)))$.
9. Negating (an Implication) means Swapping: $\neg(a \rightarrow b) \vdash (b \rightarrow a)$.
10. Distributive Law for Conjunction and Implication: $((a \rightarrow b) \wedge (a \rightarrow c)) \vdash (a \rightarrow (b \wedge c))$.
11. De Morgan's Law for Implication: $((a \wedge b) \rightarrow c) \vdash (a \rightarrow c) \vee (b \rightarrow c)$.
12. Pseudo Modus Tollens: $\{(a \rightarrow (b \rightarrow c)), \neg c, a\} \vdash \neg b$.
13. Yet Another Modus Tollens: $\{(a \rightarrow (b \rightarrow c)), \neg b, \neg c\} \vdash \neg a$.
(if you choose **prove**, do not employ Modus Tollens)
14. Cutting Rule: $\{(a \vee b), (\neg b \vee c)\} \vdash (a \vee c)$.
15. Goal Switching in Disjunction of Implications: $((a \rightarrow x) \vee (b \rightarrow y)) \vdash ((a \rightarrow y) \vee (b \rightarrow x))$.

Natural Deduction Rules (or Magnificent Seven Little Engines of Logic)

1. Conjunction Introduction $\land\mathcal{I}$ and Conjunction Eliminations $\land\mathcal{E}1$ and $\land\mathcal{E}2$:

$$\begin{array}{r|l} 1 & \alpha \\ 2 & \beta \\ \hline 3 & (\alpha \land \beta) \end{array} \qquad \begin{array}{r|l} 1 & (\alpha \land \beta) \\ \hline 2 & \alpha \end{array} \qquad \begin{array}{r|l} 1 & (\alpha \land \beta) \\ \hline 2 & \beta \end{array}$$

2. Disjunction Introductions $\lor\mathcal{I}1$ and $\lor\mathcal{I}2$, and Disjunction Elimination $\lor\mathcal{E}$ (Proof by Cases):

$$\begin{array}{r|l} 1 & \alpha \\ \hline 2 & (\alpha \lor \beta) \end{array} \qquad \begin{array}{r|l} 1 & \alpha \\ \hline 2 & (\beta \lor \alpha) \end{array} \qquad \begin{array}{r|l} 1 & (\alpha \lor \beta) \\ 2 & \quad \begin{array}{|l} \alpha \\ \hline \gamma \end{array} \\ 3 & \\ 4 & \quad \begin{array}{|l} \beta \\ \hline \gamma \end{array} \\ 5 & \\ 6 & \gamma \end{array}$$

3. Implication Introduction $\rightarrow\mathcal{I}$ and Implication Eliminations (Modus Ponens (MP) and Modus Tollens (MT)):

$$\begin{array}{r|l} 1 & \quad \begin{array}{|l} \alpha \\ \hline \beta \end{array} \\ 2 & \\ 3 & (\alpha \rightarrow \beta) \end{array} \qquad \begin{array}{r|l} 1 & (\alpha \rightarrow \beta) \\ 2 & \alpha \\ \hline 3 & \beta \end{array} \qquad \begin{array}{r|l} 1 & (\alpha \rightarrow \beta) \\ 2 & \neg\beta \\ \hline 3 & \neg\alpha \end{array}$$

4. Negation Introduction $\neg\mathcal{I}$ (Proof By Contradiction) and Negation Elimination $\neg\mathcal{E}$:

$$\begin{array}{r|l} 1 & \quad \begin{array}{|l} \alpha \\ \hline \bot \end{array} \\ 2 & \\ 3 & \neg\alpha \end{array} \qquad \begin{array}{r|l} 1 & \alpha \\ 2 & \neg\alpha \\ \hline 3 & \bot \end{array}$$

5. Falsehood Introduction $\bot\mathcal{I}$ and Falsehood Elimination $\bot\mathcal{E}$:

$$\begin{array}{r|l} 1 & \alpha \\ 2 & \neg\alpha \\ \hline 3 & \bot \end{array} \qquad \begin{array}{r|l} 1 & \bot \\ \hline 2 & \alpha \end{array}$$

6. Double Negation Introduction $\neg\neg\mathcal{I}$ and Double Negation Elimination $\neg\neg\mathcal{E}$:

$$\begin{array}{r|l} 1 & \alpha \\ \hline 2 & \neg\neg\alpha \end{array} \qquad \begin{array}{r|l} 1 & \neg\neg\alpha \\ \hline 2 & \alpha \end{array}$$

7. Law of Excluded Middle (LEM):

$$\begin{array}{r|l} 1 & (\alpha \lor \neg\alpha) \end{array}$$

Example Prove or disprove this one-half of the Distributive Law:

$$((a \vee b) \wedge c) \vdash (a \wedge c) \vee (b \wedge c).$$

Prove! Here is the derivation using Natural Deduction rules:

1	$((a \vee b) \wedge c)$	premise
2	$(a \vee b)$	$\wedge\mathcal{E}1$, 1
3	$\quad a$	assumption
4	$\quad c$	$\wedge\mathcal{E}2$, 1
5	$\quad (a \wedge c)$	$\wedge\mathcal{I}$, 3, 4
6	$\quad (a \wedge c) \vee (b \wedge c)$	$\vee\mathcal{I}1$, 5
7	$\quad b$	assumption
8	$\quad c$	$\wedge\mathcal{E}2$, 1
9	$\quad (b \wedge c)$	$\wedge\mathcal{I}$, 7, 8
10	$\quad (a \wedge c) \vee (b \wedge c)$	$\vee\mathcal{I}2$, 8
11	$(a \wedge c) \vee (b \wedge c)$	$\vee\mathcal{E}$, 2, 3–6, 7–10

So, $((a \vee b) \wedge c) \vdash (a \wedge c) \vee (b \wedge c)$ is *provable* (*derivable* or *valid*) under the Natural Deduction proof system.