Policy on homeworks

- **Collaboration**: You may discuss a question with any other student currently taking CS81 provided: (i) both of you contribute equally; (ii) you come away from any discussion with an understanding in your mind (and no archived solution of any form is retained); (iii) your submission is your own work prepared by yourself on a separate occasion.

- **Reference materials**: You should only refer to materials from this semester of CS81 (class notes, handouts, textbooks, grutors, instructor, etc).

- **Submission**: Your submission should be legible or is prepared using TeX.

Grammars

1. Consider the following context-free grammar \( G = (V, \Sigma, R, S) \), where \( V = \{ E, T, F \} \), \( \Sigma = \{ a, *, (, ) \} \), \( S = E \), and rules \( R \) defined as:

\[
E \rightarrow ET | T \\
T \rightarrow T* | F \\
F \rightarrow (E) | a
\]

(a) Draw a pushdown automata (PDA) for \( L(G) \).
(b) Convert \( G \) to Chomsky Normal Form. Show and explain your steps.
(c) Decide if \( w \in L(G) \) for \( w = a*a*a \) using the Cocke-Younger-Kasami (CYK) algorithm. If \( w \in L(G) \), show how to recover the derivation \( S \Rightarrow w \) from the dynamic programming table.
(d) Convert \( G \) to Greibach Normal Form. Show and explain your steps.
(e) Is \( G \) an ambiguous grammar?

2. An unrestricted grammar is a 4-tuple \( G = (V, \Sigma, R, S) \) where \( V \) is a finite set of variables, \( \Sigma \) is a finite alphabet, \( S \) is a start symbol, and \( R \subseteq (V \cup \Sigma)^*V(V \cup \Sigma)^* \times (V \cup \Sigma)^* \) is a set of rules.

Consider the following grammar \( G = (\{S, L, A, B, [ , ]\}, \{a, b\}, R, S) \) with the rules:

\[
S \rightarrow L ] \\
L \rightarrow La | Lb | [ \\
| a \rightarrow a | A \\
| b \rightarrow b | B \\
Ab \rightarrow aA \\
ba \rightarrow bA \\
Ba \rightarrow aB \\
Bb \rightarrow bB \\
A ] \rightarrow | a \\
B ] \rightarrow | b \\
| ] \rightarrow \epsilon
\]

(a) Determine if \( L(G) = L \) where \( L = \{ww : w \in \{a, b\}^*\} \) or not.

If \textit{yes}, explain carefully the rules of \( G \) (their specific roles or purpose, etc.).

If \textit{not}, show a counterexample of either \( w \not\in L \) (for which \( S \Rightarrow w \)) or \( w \in L \) (for which \( S \Rightarrow w \) does not hold); and then, show how to fix \( G \) so it generates \( L \).

(b) Adapt \( G \) to obtain a grammar for \( L = \{www : w \in \{a, b\}^*\} \). Explain your rules carefully.