CS81 HW#9 Spring 2012 *Out: 04/05/2012. Due: 04/11/2012 (before class)*

Policy on homeworks

- *Collaboration*: You may discuss a question with any other student currently taking CS81 provided: (i) both of you contribute equally; (ii) you come away from any discussion with an understanding in your mind (and no archived solution of any form is retained); (iii) your submission is your own work prepared by yourself on a separate occassion.
- *Reference materials*: You should only refer to materials from this semester of CS81 (class notes, handouts, textbooks, grutors, instructor, etc).
- *Submission*: Your submission should be legible or is prepared using TeX.

Grammars

1. Consider the following context-free grammar $G = (V, \Sigma, R, S)$, where $V = \{E, T, F\}$, $\Sigma = \{\mathbf{a}, *, (,)\}$, $S = E$, and rules R defined as:

$$\begin{aligned} E &\rightarrow E\ T \mid T \\ T &\rightarrow T * \mid F \\ F &\rightarrow (\ E\ ) \mid \mathbf{a} \end{aligned}$$

   (a) Draw a *pushdown automata* (PDA) for $L(G)$.

   (b) Convert G to *Chomsky Normal Form*. Show and explain your steps.

   (c) Decide if $w \in L(G)$ for $w = \mathbf{a}*\mathbf{a}*\mathbf{a}$ using the Cocke-Younger-Kasami (CYK) algorithm. If $w \in L(G)$, show how to recover the derivation $S \stackrel{*}{\Rightarrow} w$ from the dynamic programming table.

   (d) Convert G to *Greibach Normal Form*. Show and explain your steps.

   (e) Is G an *ambiguous* grammar?

2. An unrestricted *grammar* is a 4-tuple $G = (V, \Sigma, R, S)$ where V is a finite set of variables, Σ is a finite alphabet, S is a start symbol, and $R \subseteq (V \cup \Sigma)^\star V (V \cup \Sigma)^\star \times (V \cup \Sigma)^\star$ is a set of rules.

   Consider the following grammar $G = (\{S, L, A, B, [,]\}, \{a, b\}, R, S)$ with the rules.

$$\begin{aligned} S &\rightarrow L\ ] \\ L &\rightarrow La \mid Lb \mid [\\ [\ a &\rightarrow a\ [\ A \\ [\ b &\rightarrow b\ [\ B \\ Aa &\rightarrow aA \\ Ab &\rightarrow bA \\ Ba &\rightarrow aB \\ Bb &\rightarrow bB \\ A\ ] &\rightarrow]\ a \\ B\ ] &\rightarrow]\ b \\ [\ ] &\rightarrow \epsilon \end{aligned}$$

   (a) Determine if $L(G) = L$ where $L = \{ww : w \in \{a, b\}^\star\}$ or not.
   If *yes*, explain carefully the rules of G (their specific roles or purpose, etc.).
   If *not*, show a counterexample of either $w \notin L$ (for which $S \stackrel{*}{\Rightarrow} w$) or $w \in L$ (for which $S \stackrel{*}{\Rightarrow} w$ does not hold); and then, show how to fix G so it generates L.

   (b) Adapt G to obtain a grammar for $L = \{www : w \in \{a, b\}^\star\}$. Explain your rules carefully.