Collaboration Policy: You are allowed to discuss problems with others. However, the work you turn in must be by your own hand, done without transcription from other sources. If you receive significant help, cite the problem and the nature and source of the help.

1. [10] Let $L(x, y)$ mean “$x$ loves $y$”. Translate each of the following statements into a predicate logic formula. John, Mary, and Tom are constant symbols. Loving is not regarded as exclusive: If one person loves another, he/she might love others as well. By a “lover”, we mean a person who loves someone (including possibly him or herself).

   a. John loves Mary.
   b. Mary loves herself.
   c. John is a lover.
   d. John loves everyone except Mary.
   e. John loves exactly one person.
   f. Mary loves exactly two people.
   g. Tom loves no one.
   h. Tom loves everyone who does not love him.
   i. Tom loves everyone who does not love him/herself.
   j. Tom loves any and only those who do not love him/herself.

Continuing with the nomenclature above, in 2-4, check each syllogism or sequent for validity by the tableau method. If not valid, give a counterexample. (4 is a little tricky, so be careful.)

2. [10] Given
   a. Everyone loves every lover.
   b. Mary loves herself.
   It follows that
   c. Mary loves John.

3. [10] Given
   a. Tom loves everyone who does not love him/herself (and maybe others).
   It follows that
   b. Tom loves himself.

4. [10] Given
   a. Tom loves any and only those who do not love him/herself.
   It follows that
   b. Tom loves Mary.
In 5-10, check each sequent for validity by the tableau method. If not valid, give a counterexample.

5. [10] \[ \frac{((\forall x A(x)) \to (\exists x B(x))) \to (\forall x (A(x) \to B(x)))}{\text{\textit{sequent}}} \]

6. [10] \[ \frac{((\forall x A(x)) \to (\exists x B(x)))}{\frac{\frac{}{(\exists x (A(x) \to B(x)))}}{\text{\textit{sequent}}}} \]

7. [10] \[ \frac{((\exists x (A(x) \to B(x))))}{\frac{\frac{}{((\forall x A(x)) \to (\exists x B(x)))}}{\text{\textit{sequent}}}} \]


9. [10] (Huth&Ryan, exercise 2.3.11c, page 162)

\[ \exists x \exists y (H(x,y) \lor H(y,x)), \neg \exists x H(x, x) \vdash \exists x \exists y \neg (x=y) \]

10. [10] (Huth&Ryan, exercise 2.4.12k, page 164)

\[ \forall x \exists y (P(x) \to Q(y)) \vdash \exists y \forall x (P(x) \to Q(y)) \]