1. (H&R, page 300, Ex. 4.3, 5.c)  
Prove the correctness of the following program, with respect to the indicated assumption and expectation. Use the WP approach.

Assumption: $x > 1$  
Expectation: $y > 0 \land x > y$  
Program:

```
a := 1;
y := x;
y := y - a;
```

2. (H&R, page 301, Ex. 4.3, 14)  
Prove the partial correctness of Multi1:

Assumption: $y \geq 0$  
Expectation: $z = x \times y$  
Program Multi1:

```
a := 0;
z := 0;
while( a != y )
{
z := z + x;
a := a + 1;
}
```

After determining a loop invariant I, use the WP approach.

3. (H&R, page 302, Ex. 4.3, 17)  
The program Div is supposed to compute the quotient of integers $x$ by $y$, defined to be the unique integer $d$ such that there exists some integer $r$ – the remainder – with $r < y$ and $x = d \times y + r$. For example, if $x = 15$ and $y = 6$, then $d = 2$ because $15 = 2 \times 6 + 3$, where $r = 3$. Prove the partial correctness of Div:

Program: Div:

```
r := x;
d := 0;
while( r >= y )
{
r := r - y;
d := d + 1;
}
```
4. (H&R, page 303, Ex. 4.4, 1.b)
Prove the total correctness of Mult in problem 2.

5. (H&R, page 303, Ex. 4.4, 1.f)
Prove the total correctness of Div in problem 3.

6. Prove the total correctness of the program below, with respect to the assumption and expectation indicated. Use WP reasoning.

Assumption: \( n = N \land N \geq 0 \land b > 0 \)
Expectation: \( r = b^N \)

\[(r, s) := (1, b);
while( n > 0 )
\{
  if( n \% 2 == 1 )
  \{
    r := r \cdot s;
  \}
  (n, s) := (n/2, s\cdot s);
\}
\]

Here \( / \) represents integer (truncating) division and \( \% \) is the modulus function. For example, \( 5/2 = 2, 4/2 = 2, 5\%3 = 2, 6\%3 = 0, \) etc.