Adapted from exercises and problems in Sipser 3rd edition:

1.6 Give state diagrams of DFAs recognizing the following languages:

   f. [5 points] \( \{ w \in \{0, 1\}^* \mid w \text{ doesn't contain the substring } 110 \} \)

   j. [5 points] \( \{ w \in \{0, 1\}^* \mid w \text{ contains at least two 0s and at most one 1} \} \)

1.14

   a. [5 points] Show that if \( M \) is a DFA that recognizes language \( B \), swapping the accepting and non-accepting states in \( M \) yields a new DFA recognizing the complement of \( B \). Conclude that the class of regular languages is closed under complement.

   b. [5 points] Show by giving an example that if \( M \) is an NFA that recognizes language \( C \), swapping the accepting and nonaccepting states in \( M \) doesn't necessarily yield a new NFA that recognizes the complement of \( C \). Is the class of languages recognized by NFAs closed under complement? Explain your answer.

1.31 [15 points] For any string \( w = w_1w_2 \cdots w_n \) the reverse of \( w \), written \( w^R \), is the string \( w \) in reverse order, \( w_n \cdots w_2w_1 \). For any language \( A \), let \( A^R = \{ w^R \mid w \in A \} \). Show that if \( A \) is regular, so is \( A^R \).

1.31 annex [5 points] Show an example of a DFA accepting \( A^R \) where \( A \) is the language of the DFA in 1.6f above.

1.32 [20 points] Let

\[
\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \cdots, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}.
\]

\( \Sigma_3 \) contains all size 3 columns of 0s and 1s. A string of symbols in \( \Sigma_3 \) gives three rows of 0s and 1s. Consider each row to be a binary numeral and let

\[
B = \{ w \in \Sigma_3^* \mid \text{the bottom row is the sum of the top two rows} \}
\]

For example,
Show that B is regular by presenting a DFA for it. (Hint: Working with \( B^R \) is easier. You may assume the result claimed in Problem 1.31.)

1.37 [15 points] Let 
\( C_n = \{ x | x \text{ is a binary numeral that is a multiple of } n, \text{ most-significant bit first} \} \).
Show that for each \( n \geq 1 \), the language \( C_n \) is regular. You may adopt the convention that the empty string \( \varepsilon \) is in every \( C_n \).

1.37 annex [5 points] Illustrate with a DFA for 
\( C_6 = \{ \varepsilon, 0, 110, 1100, 10010, 11000, 11110, \ldots \} \).

1.57 [15 points] If \( A \) is any language, let \( \text{firsthalves}(A) \) be the set of all first halves of strings in \( A \) so that 
\[
\text{firsthalves}(A) = \{ x | \text{for some } y, |x| = |y| \text{ and } xy \in A \}.
\]
Show that if \( A \) is regular, then so is \( \text{firsthalves}(A) \)

1.57 annex [5 points] Illustrate by giving a DFA for \( \text{firsthalves}(A) \), where \( A \) is the language of the DFA in 1.6j.