1. [10 points] (Sipser 1.16b) Use the subset construction to convert the following NFA to a DFA.

```
1 --ε --> 2
  |        |
  v        v
 a       a, b
  |        |
1 --a --> 3
  |        |
  v        v
 b       b
```

2. [20 points] (after Sipser 1.19b) Construct a minimal-state (one having no two states equivalent) DFA that accepts the language represented by the regular expression \((00)^*11 \cup 01)^*\).

3. [10 points] (Sipser 1.21) Derive a regular expression for the language of the DFA below using the method described in class.

```
1 --a,b --> 2
  |        |
  v        v
 b       b
  |        |
1 --a --> 3
```

4. [15 points] Give an algorithm for solving the following problem: Given two regular expressions over the same alphabet, is there at least one string common to both of their languages.

5. [15 points] (Sipser, 1.53)

Let \(Σ = \{0, 1, +, =\}\). Show that the language \(ADD\) over \(Σ\) is not regular.

\[
ADD = \{x=y+z \mid x, y, \text{and } z \text{ are binary numerals and } x \text{ is the sum of } y+z\}.
\]

(use the Myhill-Nerode theorem or the pumping lemma)
6. [15 points] If the following language is regular, derive a regular expression for it. Otherwise, prove it is not regular.

\{ x \in \{0, 1\}^* | \text{For every prefix } y \text{ of } x | |\#_0(y) - \#_1(y)| \leq 2 \}

Here \( \mid \mid \) represents absolute value and \( \#_{\sigma}(y) \) means the number of occurrences of \( \sigma \) in \( y \).

7. [15 points] Show that the language

\( PRIMES = \{ 1^p \mid p \text{ is prime} \} = \{11, 111, 11111, 1111111, \ldots \} \)

is not regular.