Computer Science 81, Fall 2013
Assignment 12 of 12
Due Wednesday December 11
Automatic Extension to December 12 at 9 PM (No further extensions)

1. [10 points] (after Sipser 4.4) Let $A_{\epsilon_{\text{CFG}}} = \{ (G) \mid G \text{ is a CFG that generates } \epsilon \}$.
   Show that $A_{\epsilon_{\text{CFG}}}$ is decidable by giving an algorithm for deciding it. Apply the algorithm to these two grammars over terminal alphabet \{a, b\} with start symbol $S$:
   \[
   \begin{align*}
   \text{G1:} & \quad S \rightarrow AB \\
   & \quad A \rightarrow \epsilon \\
   & \quad B \rightarrow SC \mid a \\
   \text{G2:} & \quad S \rightarrow AB \\
   & \quad A \rightarrow B \mid \epsilon \\
   & \quad B \rightarrow AC \\
   & \quad C \rightarrow \epsilon \mid a
   \end{align*}
   \]

2. [10 points] (after Sipser 4.11) Let $\text{INFINITE}_{\text{CFG}} = \{ (G) \mid G \text{ is a CFG and } L(G) \text{ is an infinite language} \}$. Show that $\text{INFINITE}_{\text{CFG}}$ is decidable by giving an algorithm for deciding it. Apply the algorithm to the two grammars in the preceding problem.

3. [15 points] (Sipser 4.30) Let $A$ be a Turing-recognizable language consisting of descriptions of Turing machines, \{ $\langle M_1 \rangle$, $\langle M_2 \rangle$, $\ldots$ \}, where every $M_i$ is a decider. Prove that some decidable language $D$ is not decided by any decider $M_i$, the description of which appears in $A$. (Hint: You may find it helpful to consider an enumerator for $A$.) Further hint: Identify the set of all strings over the alphabet of $A$ with the natural numbers, enabling the use of a string as an index into $A$.

4. [5 points] (Sipser 5.1) Show that $\text{EQ}_{\text{CFG}}$ (the equality problem for context-free grammars) is undecidable.

5. [10 points] (Sipser 5.2) Show that $\text{EQ}_{\text{CFG}}$ is co-recognizable.

6. [5 points] (Sipser 5.2) Find a match in the following instance of the Post Correspondence Problem:
   \[
   \left\{ \begin{array}{c}
   \left[ \frac{ab}{a} \right], \left[ \frac{b}{b} \right], \left[ \frac{aba}{a} \right], \left[ \frac{aa}{b} \right] \end{array} \right\}
   \]

7. [10 points] (Sipser 5.9) Let $T = \{ (M) \mid M \text{ is a TM that accepts } w^R \text{ if it accepts } w \}$. Show that $T$ is undecidable (without appealing to Rice’s theorem). Hint: Reduce $A_{\text{TM}}$ to $T$ by showing how to transform $\langle M, x \rangle$ into $\langle M' \rangle$ so that $x \in L(M)$ iff $\langle M' \rangle \in T$. If $M'$ accepts only one string $w$ but does not accept $w^R$, then $\langle M' \rangle \notin T$. 
8. [5 points] (Sipser 5.17) Show that the Post Correspondence Problem is decidable over the unary alphabet $\Sigma = \{1\}$.

9. [10 points] (Sipser 5.22) Show that A is Turing-recognizable iff $A \leq_m A_{TM}$.

10. [10 points] Determine whether or not language WNB below is decidable. Assume the tape alphabet is \{blank, 0, 1\}. Prove your answer.

    WNB = \{<M> | M writes a non-blank symbol when started on a blank tape\}

11. [10 points] Let $\text{INFINITETM} = \{\langle M \rangle | M \text{ is a TM and } L(M) \text{ is an infinite language}\}$. Is decidable, recognizable, corecognizable, or neither? Justify your answers.