Fun with Interpretations

• A 2-ary predicate represents the binary relation in an interpretation, i.e. a set of pairs of domain elements.

• Various properties of relations can be expressed using predicate logic formulas.

• In the following, what formula characterizes each relation represented by predicate L for “loves”, and possibly the predicate = for equality.
Everyone loves someone.

Formula: $\forall x \exists y \ L(x, y)$
Everyone is loved by someone.

Formula:
Someone loves everyone.

Formula:

“Pollyanna”
No one loves everyone.

Formula:
No one loves someone.

Formula:
Everyone loves everyone.

“Commune”

Formula:

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Everyone loves exactly one.

Formula:
Everyone loves him/herself.

“Reflexive”

Formula:
Everyone loves him/herself and only him/herself

“Narcissists’ Convention”

Formula:
Symmetric

Formula:
Transitive

Formula:
Equivalence Relation

Reflexive, symmetric, transitive

Formula:
Antisymmetric

Formula:
Partial Order ("poset").

Reflexive, Antisymmetric, Transitive

Formula:
Linear Order

Formula:
Translate to Predicate Logic

- If someone loves someone, then the latter loves him/herself.
- No one loves more than one other.
- If someone is loved by at least two others, then everyone loves the former.
- Challenge yourself with some of your own.