Harvey Mudd College, Computer Science 81, Spring 2013

Assignment 1

Propositional Natural Deduction Proofs

Due. Wed., 30 January 2011, 11:59 PM

on submission site http://www.cs.hmc.edu/~submissions as a PDF: a01.pdf

The problems in this set should be worked individually. You may get help, but transcribing solutions from someone else is not allowed.

Although you are only required to turn in these proofs, it is strongly recommended that you do others as well to acquire proficiency.

Prove each sequent using natural deduction. Use Fitch diagrams, except on 4.

1. $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

2. $(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$

3. $\vdash ((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$

4. $(p \vee q) \rightarrow r \vdash (p \rightarrow r) \wedge (q \rightarrow r)$ [Use a tree diagram for this one.]

5. $\neg p, \neg q \vdash \neg (p \vee q)$

6. $\neg (p \wedge q) \vdash \neg p \vee \neg q$

7. Recall that for sets, we have the following meanings:

   1. $A \subseteq B$ means:(for arbitrary x) $x \in A \rightarrow x \in B$

   2. $C = A \cup B$ means:(for arbitrary x) $x \in C \leftrightarrow (x \in A \vee x \in B)$

   3. $C = A \cap B$ means:(for arbitrary x) $x \in C \leftrightarrow (x \in A \wedge x \in B)$

   Use natural deduction to show: $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$
   by treating $x \in A$ etc. as propositions.

8-10. Show that, given only **one** of the rules RAA (reductio ad absurdum), LEM (law of the excluded middle), DNE (double not elimination), the other two rules can be derived using only intuitionistic rules in addition. [Only three derivations are needed: RAA $\Rightarrow$ LEM, LEM $\Rightarrow$ DNE, and DNE $\Rightarrow$ RAA.]