CS81 **Assignment 2**

Due Wednesday, 6 February 2013

1. If the formula $(p \rightarrow q) \vee (q \rightarrow r)$ is a tautology, provide a natural deduction proof of it. If not, give a counterexample valuation.

2. Determine whether or not the following entailment is valid: $(q \rightarrow (p \vee (q \rightarrow p))) \vee \neg(p \rightarrow q) \models p$

3. Suppose that Γ and Δ are sets of propositional formulas. For each statement below, if the statement is true, prove it. If it is false, give a counter-example.
   a. If Γ is satisfiable and $\Gamma \subseteq \Delta$, then Δ is satisfiable.
   b. If Γ is satisfiable and $\Delta \subseteq \Gamma$, then Δ is satisfiable.

4. Show that $\Gamma \models \psi$ iff $\Gamma \cup \{\neg\psi\} \models \bot$. To what natural deduction rules is this statement a counterpart?

5. Give a natural deduction proof ***in contextual form*** for
   $((p \rightarrow q) \wedge (q \rightarrow r)) \vdash (p \rightarrow r)$

6. Prove the part of the induction step of the **soundness theorem**, as discussed in the lecture slides, for the case of the **Not-Introduction rule**.

7. For the tautology $(p \rightarrow q) \vee (q \rightarrow p)$, show the proof that would be constructed in the structural induction part of the proof of the **completeness theorem** given in the lecture slides.

8. Use mathematical induction to prove steps 1 and 3 on slide 39 of the lecture under Proof of Completeness.