1. If the formula \((p \rightarrow q) \lor (q \rightarrow r)\) is a tautology, provide a natural deduction proof of it. If not, give a counterexample valuation.

2. Determine whether or not the following entailment is valid: \((q \rightarrow (p \lor (q \rightarrow p))) \lor \neg(p \rightarrow q) \models p\)

3. Suppose that \(\Gamma\) and \(\Delta\) are sets of propositional formulas. For each statement below, if the statement is true, prove it. If it is false, give a counter-example.
   a. If \(\Gamma\) is satisfiable and \(\Gamma \subseteq \Delta\), then \(\Delta\) is satisfiable.
   b. If \(\Gamma\) is satisfiable and \(\Delta \subseteq \Gamma\), then \(\Delta\) is satisfiable.

4. Show that \(\Gamma \models \psi\) iff \(\Gamma \cup \{\neg \psi\} \models \bot\). To what natural deduction rules is this statement a counterpart?

5. Give a natural deduction proof in contextual form for \(((p \rightarrow q) \land (q \rightarrow r)) \models (p \rightarrow r)\)

6. Prove the part of the induction step of the soundness theorem, as discussed in the lecture slides, for the case of the Not-Introduction rule.

7. For the tautology \((p \rightarrow q) \lor (q \rightarrow p)\), show the proof that would be constructed in the structural induction part of the proof of the completeness theorem given in the lecture slides.

8. Use mathematical induction to prove steps 1 and 3 on slide 39 of the lecture under Proof of Completeness.