CS81 **Assignment 3**
Sequent Calculus, Tableaux, and Predicate Calculus
Due Wednesday, 13 February 2013

Prove the sequents below using the sequent calculus, or give a counterexample if the statement is not provable.

1. $(P \rightarrow Q) \wedge (Q \rightarrow R),\ \neg R \ \vdash\ \neg P$

2. $\vdash (P \rightarrow Q) \vee ((Q \rightarrow R) \wedge (R \rightarrow S))$

Using the tableau or block tableau method, determine whether or not the sequents below are valid. For any that are not, derive *from the tableau* a valuation that falsifies the statement.

3. $\vdash\ (R \rightarrow S) \rightarrow ((P \rightarrow R) \rightarrow (P \rightarrow S))$

4. $(P \rightarrow Q), (R \rightarrow S)\ \vdash\ (P \vee R) \rightarrow (Q \wedge S)$

For the following sequent, give a proof in the form of a block tableau and demonstrate the correspondence with the sequent calculus proof in problem 1.

5. $(P \rightarrow Q) \wedge (Q \rightarrow R), \neg R \vdash\ \neg P$

Give natural deduction proofs for the sequents below. For number 6, restate the proof using English.

6. $\exists x \neg R(x)\ \vdash\ \neg \forall y R(y)$

7. $\exists x(A(x) \rightarrow B(x)) \vdash (\forall x A(x)) \rightarrow (\exists x B(x))$

8. $\neg \forall x \neg R(x) \vdash\ \exists y R(y)$