CS81 **Assignment 5**
Due Wednesday, 27 February 2013

1. [20] Let L(x, y) mean “x loves y”. Translate each of the following statements into a predicate logic formula. John, Mary, and Tom are constant symbols.

    a. John loves Mary.
    b. Mary loves herself.
    c. John is a lover (meaning there is someone John loves).
    d. John loves everyone except Mary.
    e. John loves exactly one person.
    f. Mary loves exactly two people.
    g. Tom loves no one.
    h. Tom loves everyone who does not love him back (and maybe others).
    i. Tom loves everyone who does not love him/herself (and maybe others).
    j. Tom loves any and only those who do not love him/herself.

In 2-4, check each syllogism or sequent for validity by the **tableau method**. If not valid, give a counterexample. Assume that “a lover” means a person who loves someone. That is, if L(x, y) means “x loves y”, then “x is a lover” is the same as ∃y L(x, y).

2. [10] Given
    a. Everyone loves every lover.
    b. Mary loves herself.

    It follows that
    c. Mary loves John.

3. [10] Given
    a. Tom loves everyone who does not love him/herself (and maybe others).

    It follows that
    b. Tom loves himself.

4. [10] Given
    a. Tom loves any and only those who do not love him/herself.

    It follows that
    b. Tom loves Mary.

    (This is slightly tricky, so be careful.)
    (continued)

In 5-6, check each sequent for validity by the tableau method. If not valid, give a counterexample.

5. [10] $\vdash$ $((\forall x\ A(x)) \rightarrow (\exists x\ B(x))) \rightarrow (\forall x\ (A(x) \rightarrow B(x)))$

6. [10] $((\forall x\ A(x)) \rightarrow (\exists x\ B(x))) \vdash (\exists x\ (A(x) \rightarrow B(x)))$

In 7-8, prove by the Sequent Calculus, or give a counterexample.

7. [10] $(\exists x\ (A(x) \rightarrow B(x))) \vdash ((\forall x\ A(x)) \rightarrow (\exists x\ B(x)))$

8. [10] $(\forall x A(x)) \rightarrow \exists x B(x) \vdash \exists x(A(x) \rightarrow B(x))$

9. [10] (Huth&Ryan, exercise 4.6.5) Use Hoare logic to prove the partial correctness of the following division program. All variables are of type integer.

```
{y > 0} assumption

r = x;
d = 0;
while( r >= y)
        {
        r = r – y;
        d = d + 1;
        }

{x = d*y + r ∧ r < y} expectation
```

Start by finding an appropriate loop invariant. Show the triples as a numbered list, with justifications for each. Also show the assertions as in-line comments in the program. Tabulate any assumptions you are making about integer arithmetic.