Computer Science 81, Spring 2013
Assignment 8
Due Tue. April 9

1. [10 points] Sipser book, Exercise 2.2:

2.2 **a.** Use the languages $A = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n \mid m, n \geq 0\}$ and $B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^m \mid m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

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2. [15 points] Sipser book, Exercise 2.13:

2.13 Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{\mathtt{0}, \mathtt{\#}\}$; and R is the set of rules:

$$S \rightarrow TT \mid U$$
$$T \rightarrow \mathtt{0}T \mid T\mathtt{0} \mid \mathtt{\#}$$
$$U \rightarrow \mathtt{0}U\mathtt{00} \mid \mathtt{\#}$$

a. Describe $L(G)$ in English.

b. Prove that $L(G)$ is not regular.

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3. [30 points]

Give a context free grammar that generates the language $\{x \in \{0, 1\}^* \mid \#_0(x) = \#_1(x)\}$. **Prove your answer.** (Show that your grammar generates every such string, and also that it generates only such strings. This may require a separate proof for *each direction* of set inclusion.)

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4. [15 points] Sipser book, Problem 2.10, except give a **formal** description as well.

2.10 Give an informal description of a pushdown automaton that recognizes the language A in Exercise 2.9.

$$A = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

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5. [10 points] Sipser book, Problem 2.14:

2.14 Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \rightarrow BAB \mid B \mid \varepsilon$$
$$B \rightarrow \mathtt{00} \mid \varepsilon$$

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6. [5 points] Sipser book, Problem 2.15:

2.15 Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G'. This grammar is supposed to generate A^*.

7. [15 points] Sipser book, Problem 2.31:

2.31 Let B be the language of all palindromes over $\{0,1\}$ containing an equal number of 0s and 1s. Show that B is not context free.