1. [20 points] Using JFLAP, program a Turing machine that accepts the following language: \( \{ww^R \mid w \in \{0, 1\}^* \land \#_0(w) = \#_1(w)\} \). Include a screenshot of the state diagram and one of the machine test on the following 24 inputs (using the run-multiple feature).

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<tr>
<td>( \varepsilon )</td>
<td>0101</td>
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<tr>
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<td>1001</td>
<td>11000011</td>
<td>10101101101101011010110101</td>
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Below, all proofs are informal and intended to develop intuition and expository style, so please provide convincing write-ups of each.

2. [10 points] Sipser book, Problem 3.15 c (star) only:

3.15 Show that the collection of decidable languages is closed under the operation of

- a. union.
- b. concatenation.
- c. star.
- d. complementation.
- e. intersection.

3. [10 points] Sipser book, Problem 3.16 b (concatenation) only:

3.16 Show that the collection of Turing-recognizable languages is closed under the operation of

- a. union.
- b. concatenation.
- c. star.
- d. intersection.

4. [20 points] Sipser book, Problem 3.18: Show that a language is decidable iff some enumerator enumerates the language in standard string order.
5. [10 points] Sipser book, Exercise 4.2:

4.2 Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.

6. [10 points] Sipser book, Problem 4.11:

Let \( \text{INFINITE}_{\text{PDA}} = \{ \langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language} \} \). Show that \( \text{INFINITE}_{\text{PDA}} \) is decidable.

7. [20 points] Construct a proof of Church's Theorem: There is no algorithm for determining whether a first-order predicate calculus formula is valid.

Do this by showing how to translate a Turing machine with initial and halting state into a set of clauses such that the set of clauses is unsatisfiable iff the Turing machine halts on the given initial state. (Create a clause for each TM rule and a few extra clauses for situations where the end of the tape is reached.) We show in class that the latter problem is not decidable.

Extra credit [up to 30 points]: Submit a demonstration using Prover9 or Prolog.