Computer Science 81, Spring 2013
Non-Assignment 11
Not to turn in, but helpful to work before the final

All proofs are informal and intended to develop intuition and expository style, so please provide convincing write-ups of each.

1. **[5 points]** What is the smallest alphabet Σ such that all Turing machines can be encoded as members of Σ^*?

2. **[20 points]** Consider the language of Turing machine descriptions $L = \{\langle M\rangle \mid M \text{ accepts at least 999 strings}\}$. Is L decidable, recognizable, or neither? Prove your answer.

3. **[15 points]** Consider the language of Turing machine with tape descriptions $L = \{\langle M, w\rangle \mid M \text{ accepts } w \text{ using at most 999 tape cells}\}$. Show that L is decidable. (Hint: For how many steps can M run without either accepting, rejecting, using more than 999 cells or going into an infinite loop?)

4. **[20 points]** Consider the language of Turing machine descriptions $L = \{\langle M\rangle \mid M \text{ accepts all strings of length 999 or longer}\}$. Is L decidable, recognizable, or neither? Prove your answer.

5. **[15 points]** Show that the special case of PCP (Post's Correspondence Problem) over a 1-letter alphabet is decidable.

Problems involving mapping reducibility $\leq_m$

6. **[10 points]** Suppose $R \subset \Sigma^*$ is a regular language other than $\varnothing$ and Σ^*. Let $L \subseteq \Sigma^*$ be any decidable language. Show that $L \leq_m R$ (L is mapping-reducible to R).

7. **[5 points]** Regarding the previous problem, is the same reduction true if L is recognizable but not decidable? Why or why not?

8. **[10 points]** Show that a language L is recognizable iff $L \leq_m A_{TM}$ (the acceptance language for Turing machines).