Blocks World Logic

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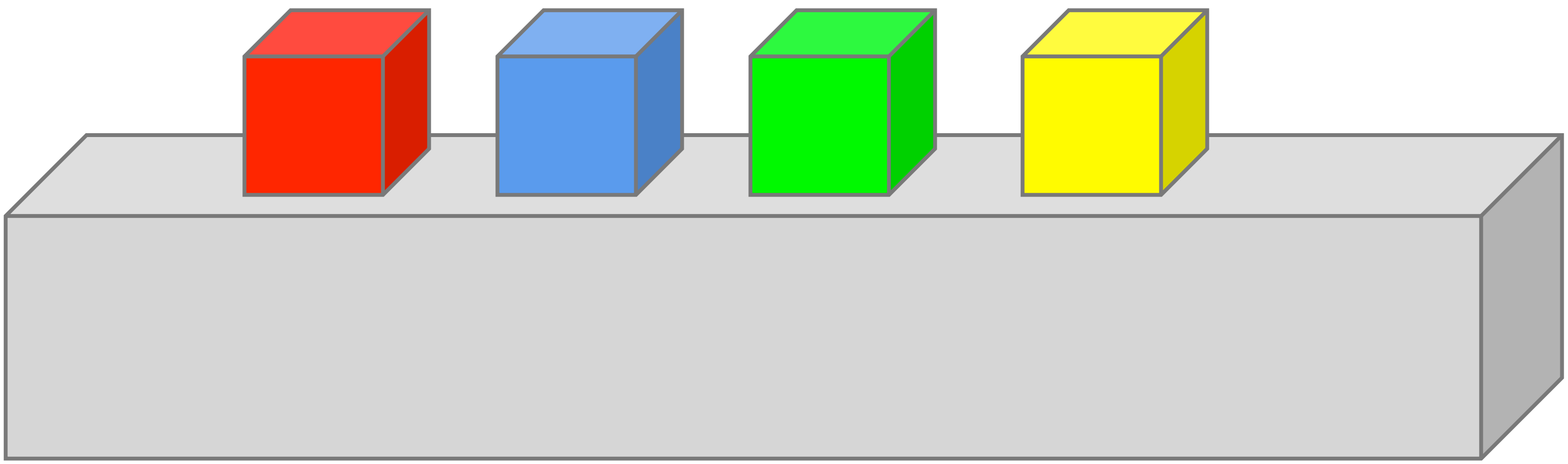
Table-Top Universes

- Blocks are identified by their color (**red**, **blue**, **green**, **yellow**, etc), except for
- **table**, which is considered to be one huge block.
- Different universes may contain different blocks.
- **table** will be in every universe.



The *on* predicate

on(x, y) means block x is on top of block y

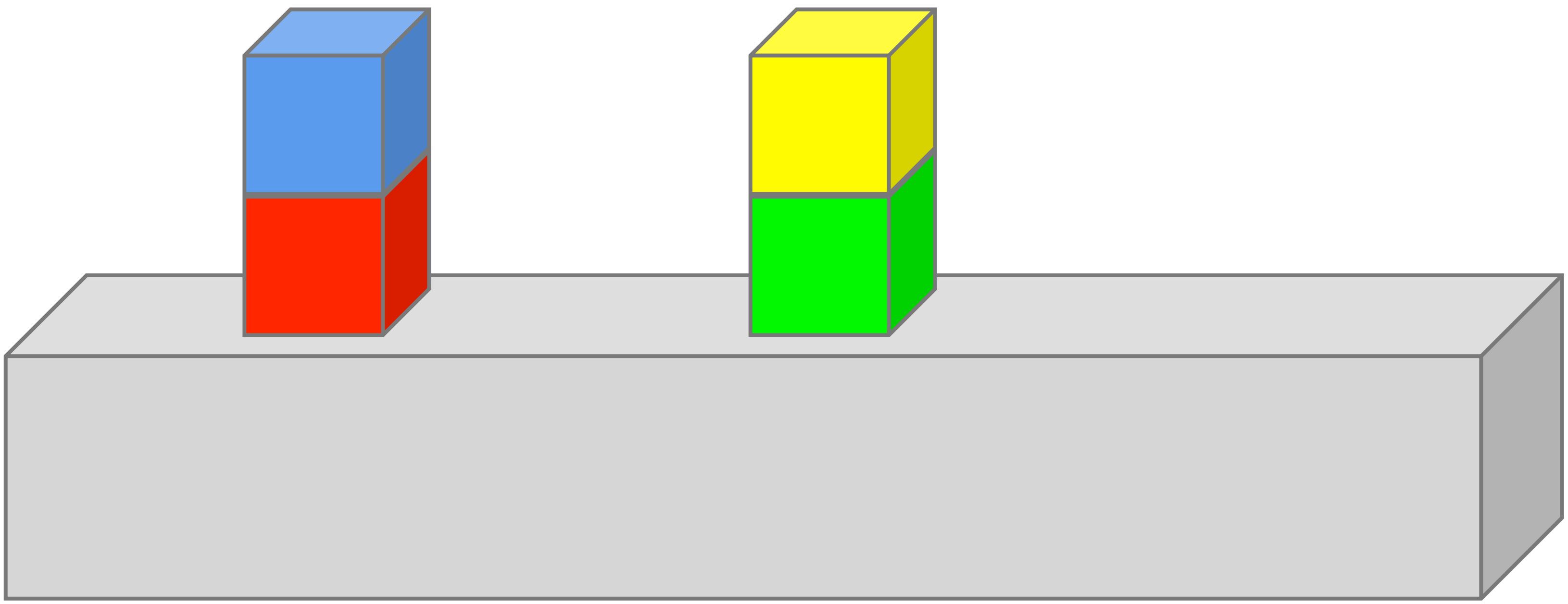
In this universe:

on(blue, red)

on(red, table)

on(yellow, green)

on(green, table)



The Blocks Language BL

- The blocks language BL is a first-order language.
- It will contain a 2-ary predicate symbol o.
- It will contain constants: b, g, r, t, y
- Other symbols will be introduced later.

Atomic Formulas in BL

Examples of atomic formulas:

o(b, r)

o(r, b)

o(y, g)

o(r, t)

because o is a 2-ary predicate symbol, b, g, r, t, y are constant symbols

Non-Atomic Formulas

Examples of non-atomic formulas:

$o(b, r) \vee o(r, b)$

$o(y, g) \wedge o(r, b)$

$o(y, g) \rightarrow o(r, t)$

$\neg o(b, t)$

Interpretations

- Each interpretation of a BL formula consists of (Δ, μ), where

  Δ is a blocks world universe
  (set of blocks)

  μ associates elements, predicates, etc. with symbols in BL

Example

- Formula: o(b, r)

- An interpretation I_1:

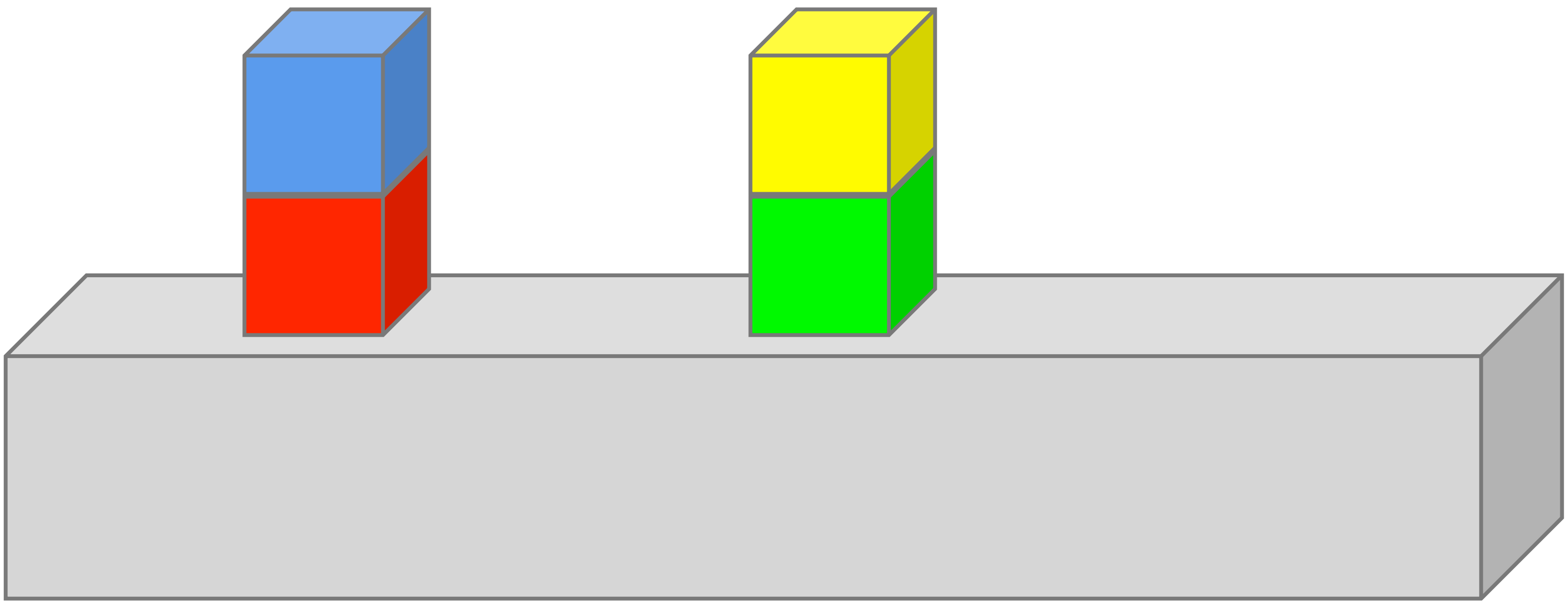
  Δ = {blue, green, red, table, yellow}

  μ(b) = blue, μ(g) = green,

  μ(r) = red, μ(t) = table, μ(y) = yellow

  μ(o) = on, where on is defined
  by the following picture

$$
\begin{aligned}
I_1[o(b, r)] &= \mu(o)(I_1[b], I_1[r]) \\
&= on(I_1[b], I_1[r]) \\
&= on(\mu(b), \mu(r)) \\
&= on(blue, red) \\
&= T \text{ (true)}
\end{aligned}
$$



Temporary convention

- For now, we will assume that each interpretation has:

  Δ = {blue, green, red, table, yellow}

  μ(b) = blue, μ(g) = green,
  μ(r) = red, μ(t) = table, μ(y) = yellow

but we may have a different μ(o)

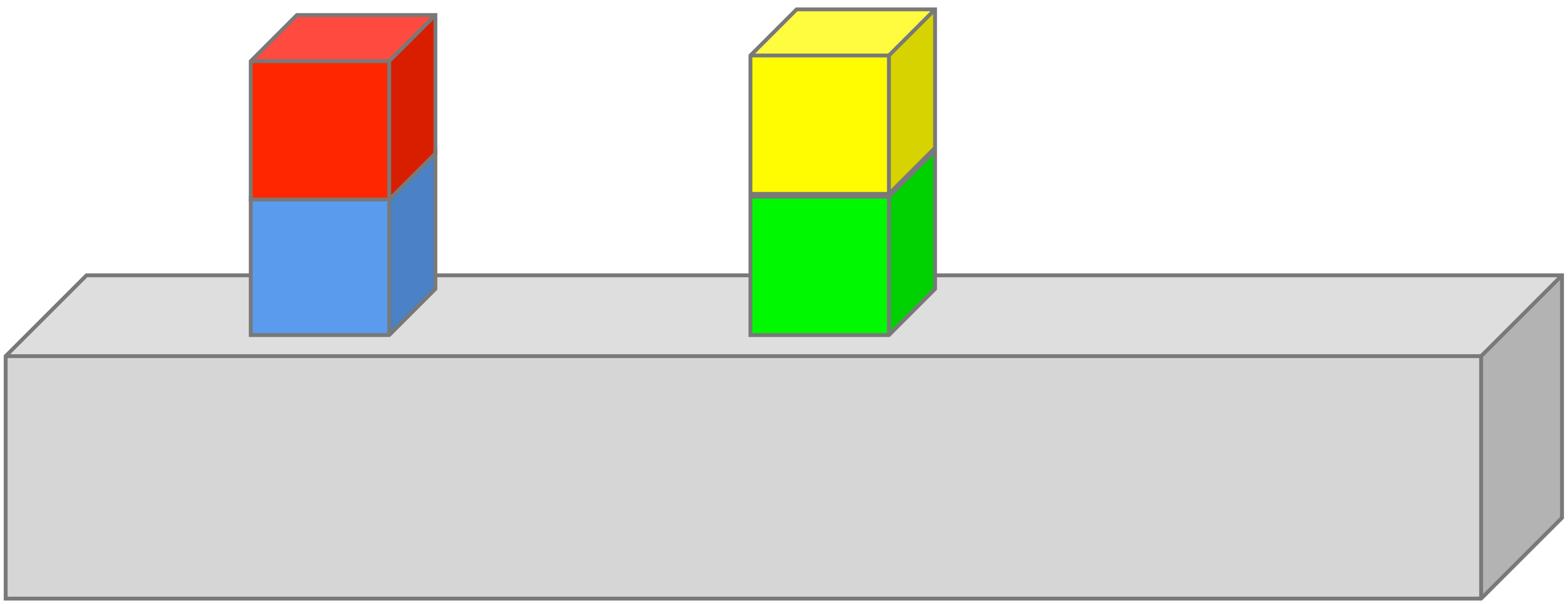
$I_2[o(b, r)] =$

$I_2[o(g, y)] =$

$I_2[o(g, y) \rightarrow o(b, r)] =$

$I_2[\forall u \forall v\ \neg(o(u, v) \wedge o(v, u))] =$

I_2 is:



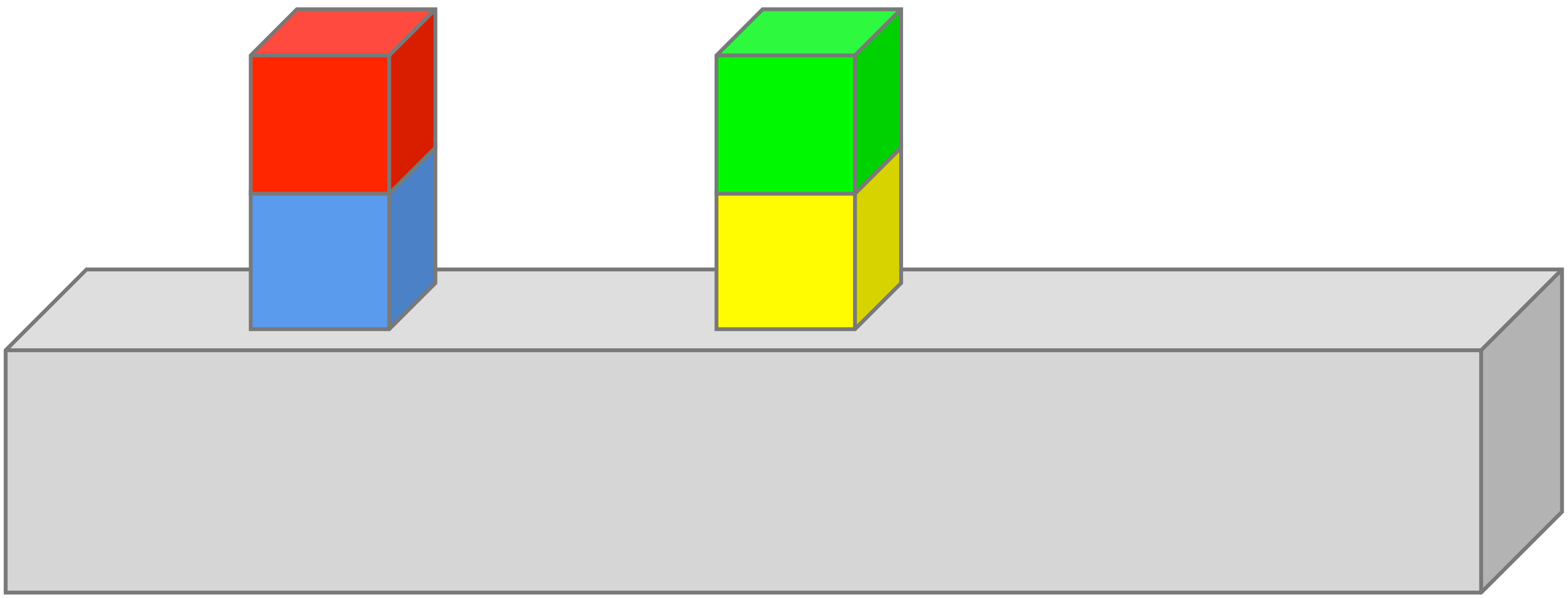
$I_3[o(b, r)] =$

$I_3[o(g, y)] =$

$I_3[o(g, y) \rightarrow o(b, r)] =$

$I_3[\forall u \forall v \ \neg(o(u, v) \wedge o(v, u))] =$

I_3 is:



Closed Formula + Interpretation gives Truth Value

- What if formula is not closed, e.g.

  $\neg(o(u, v) \wedge o(v, u))$

- If the formula is being considered in isolation, we close it by prepending $\forall u \forall v$.

- The meaning is as if the variables were $\forall u \forall v$ quantified.

Equality

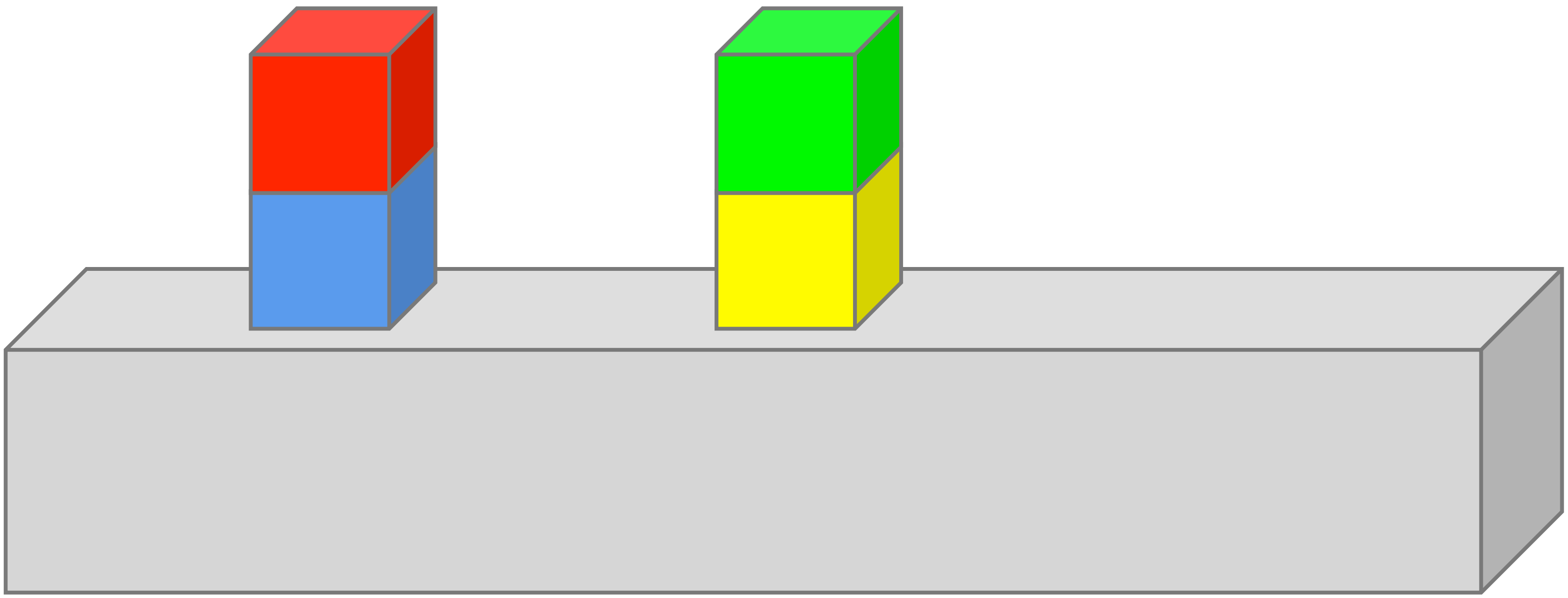
- Equality = is a 2-ary predicate symbol.

- **By convention**, for every interpretation, $\mu(=)$ is the **identity predicate** on the universe Δ.

- That is, $\mu(=)$ = {(blue, blue), (green, green), (red, red), (table, table), (yellow, yellow)}

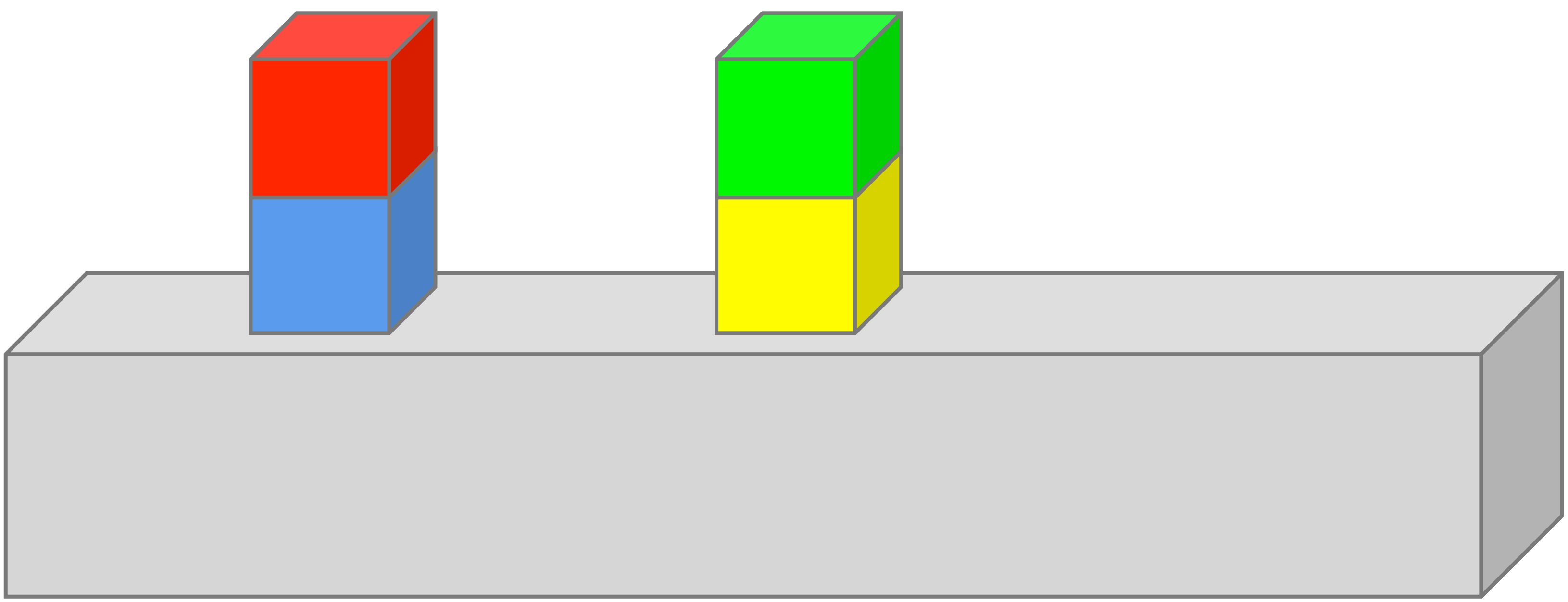
$$I_3[(o(r, x) \rightarrow (x = b)] =$$
$$I_3[(o(x, y) \rightarrow (x = g)] =$$

I_3 is:



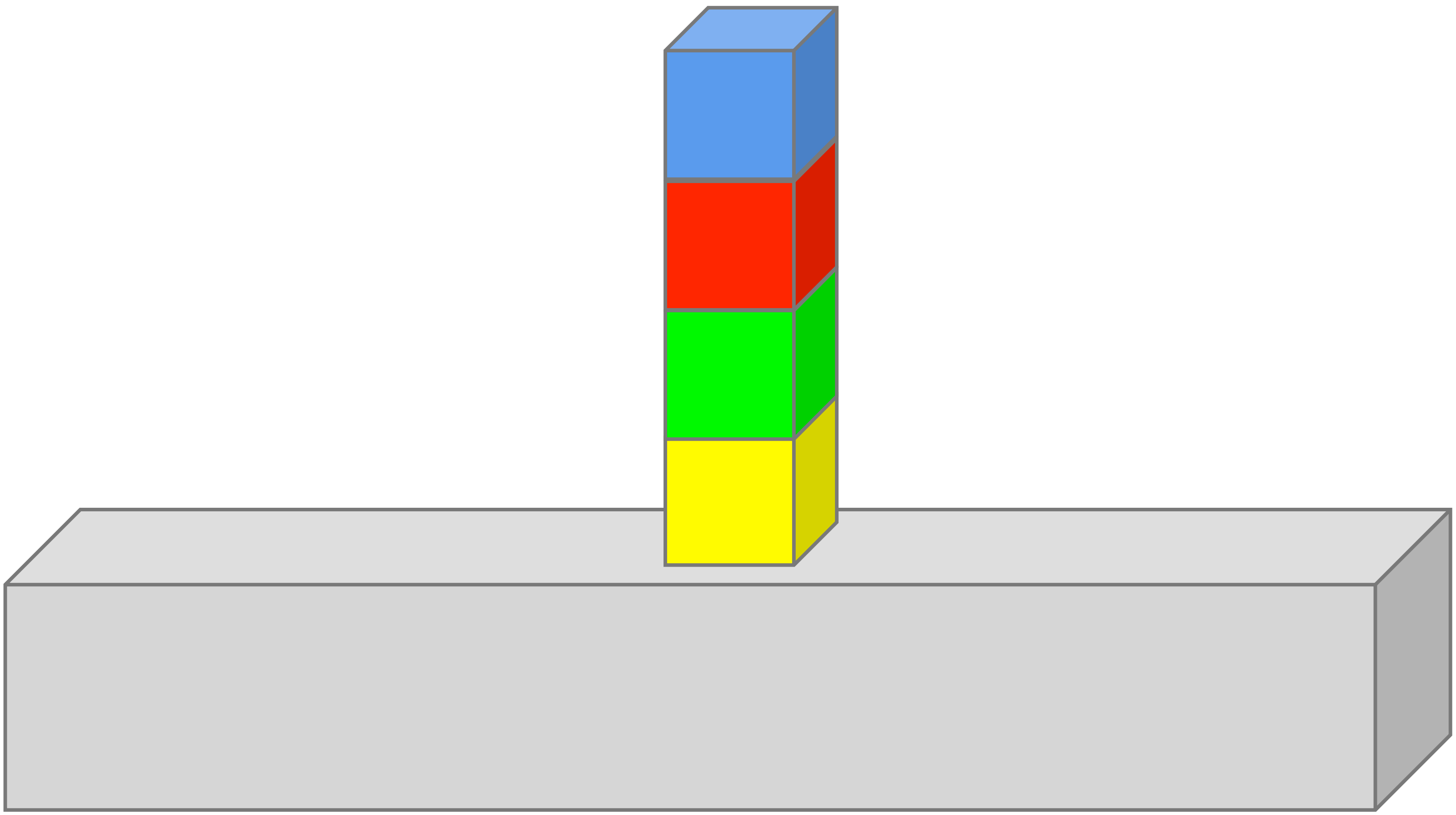
$I_3[\exists x\ o(r, x)] =$
$I_3[(\exists x\ o(x, g)] =$

I_3 is:



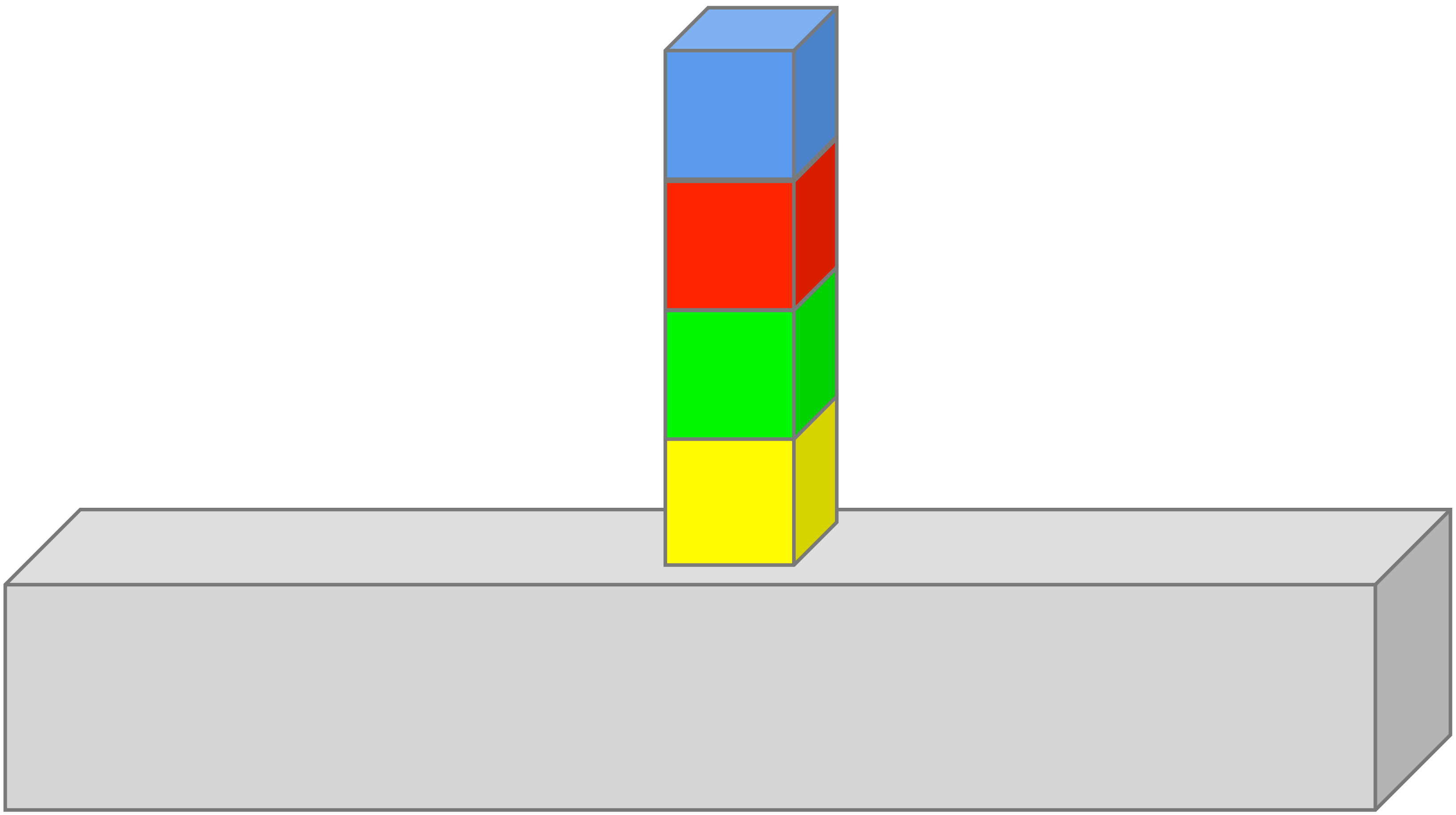
$I_4[\forall u\ \exists x\ o(x, u)] =$

I_4 is:



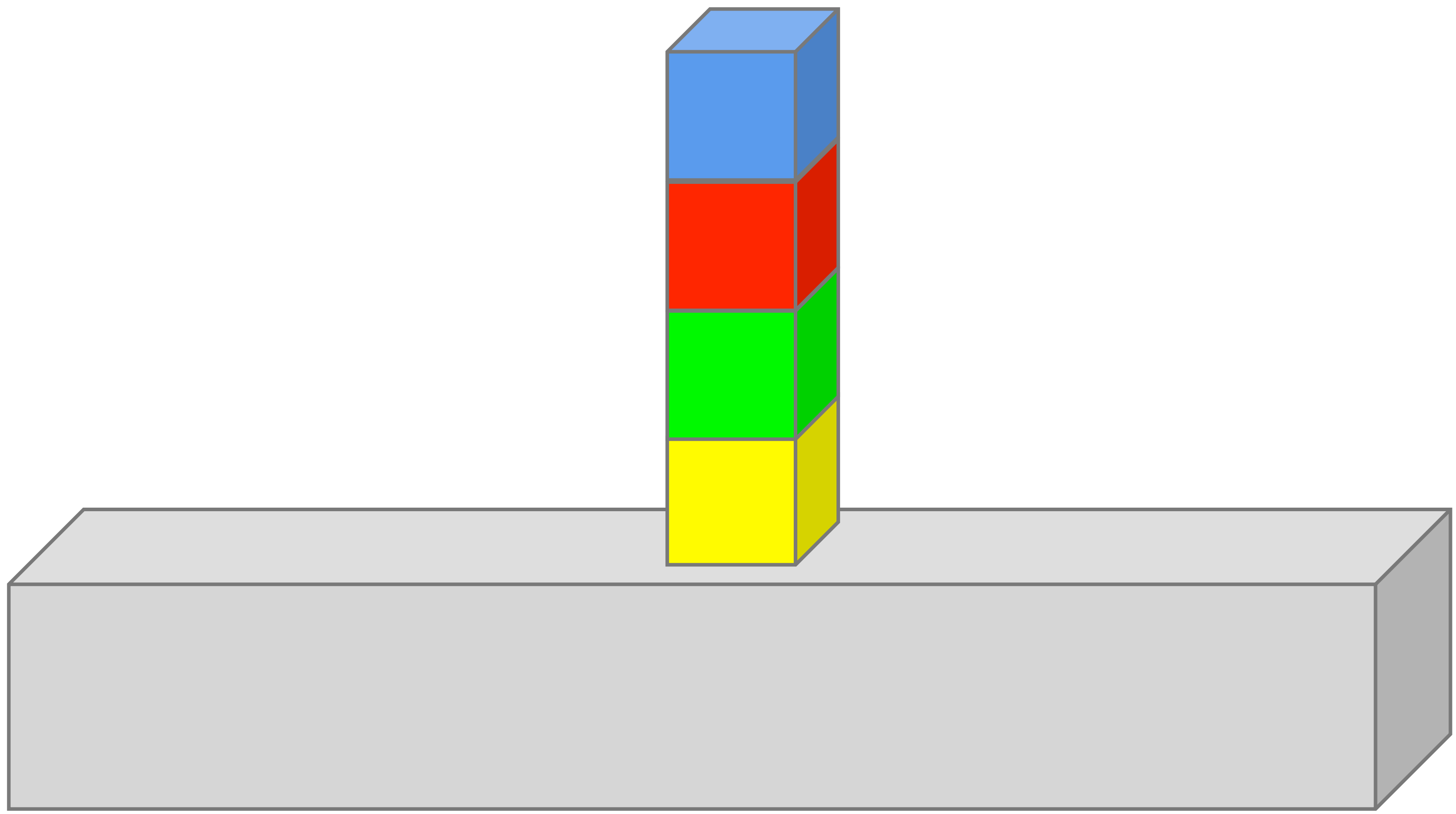
$I_4[\forall u \exists x\ o(x, u) \lor u = b\ ] =$

I_4 is:



$I_4[\forall u \exists x\ o(u, x)] =$

I_4 is:



Devise Some **Universal** Formulas for Blocks Interpretations

- By universal, I mean a formula that is true in every blocks world interpretation
- You are allowed to add new predicates

- Example:

  $\forall u \, \exists x \; o(u, x) \vee (u = t)$

  because every block is either on another block, or on the table.

More Examples

- $\forall x \; \neg o(x, x)$

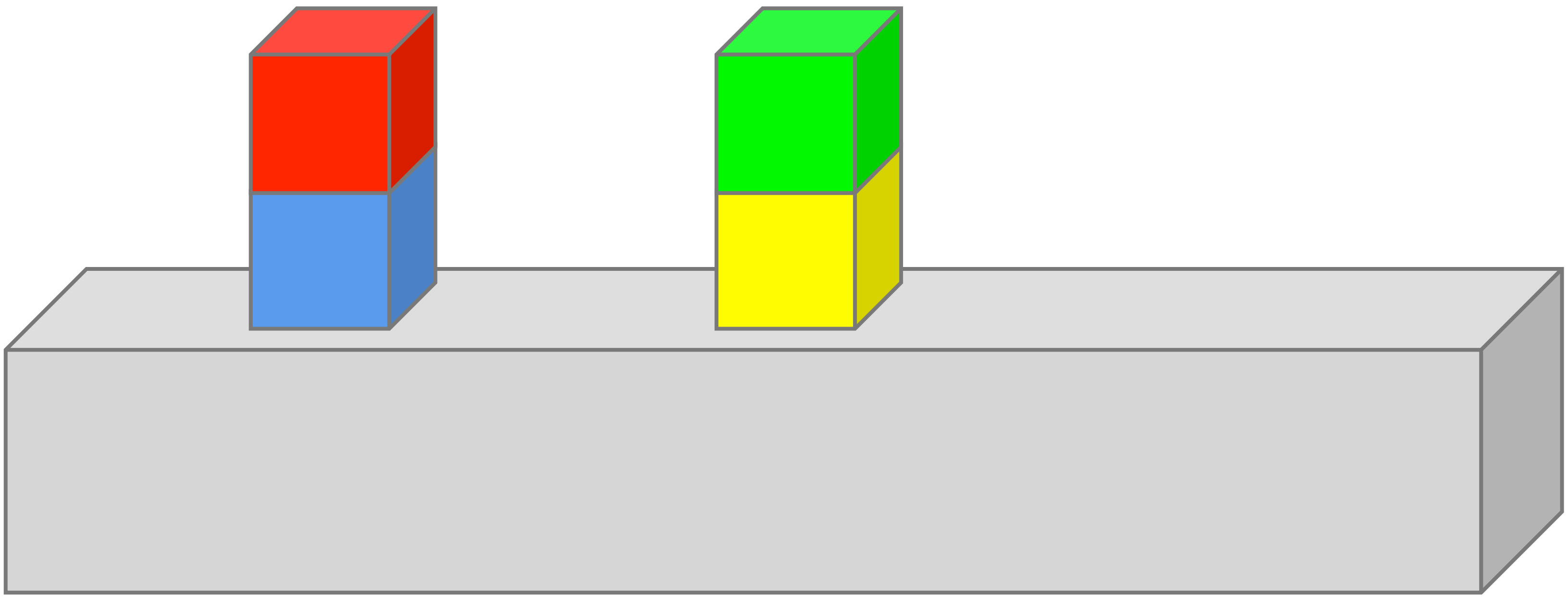
New predicate **a**

- a(u, x) means u is "above" x.

New predicate **a**

a(u, x) means u is “above” x

$I_3[(a(r, b)] = T$ $I_3[(a(r, y)] = T$



Now for blocks world or any other

- Devise some formulas that are true, regardless of the interpretation.

- These are called **valid** or **universally valid**.

- Example: $(\forall u\ o(u, u)) \rightarrow (\forall u\ \exists x\ o(u, x))$