Blocks World Logic

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Table-Top Universes

- Blocks are identified by their color (red, blue, green, yellow, etc), except for
- table, which is considered to be one huge block.
- Different universes may contain different blocks.
- table will be in every universe.
The *on* predicate

\( \text{on}(x, y) \) means block \( x \) is on top of block \( y \)

In this universe:

- \( \text{on}(\text{blue, red}) \)
- \( \text{on}(\text{yellow, green}) \)
- \( \text{on}(\text{red, table}) \)
- \( \text{on}(\text{green, table}) \)
The Blocks Language BL

• The blocks language BL is a first-order language.

• It will contain a 2-ary predicate symbol o.

• It will contain constants: b, g, r, t, y

• Other symbols will be introduced later.
Atomic Formulas in BL

Examples of atomic formulas:

- $o(b, r)$
- $o(r, b)$
- $o(y, g)$
- $o(y, g)$
- $o(r, t)$

because $o$ is a 2-ary predicate symbol, $b$, $g$, $r$, $t$, $y$ are constant symbols
Non-Atomic Formulas

Examples of non-atomic formulas:
- $o(b, r) \lor o(r, b)$
- $o(y, g) \land o(r, b)$
- $o(y, g) \rightarrow o(r, t)$
- $\neg o(b, t)$
Interpretations

• Each interpretation of a BL formula consists of $(\Delta, \mu)$, where
  
  $\Delta$ is a blocks world universe
  (set of blocks)
  
  $\mu$ associates elements, predicates, etc. with symbols in BL
Example

• Formula: $o(b, r)$

• An interpretation $I_1$:
  $\Delta = \{\text{blue, green, red, table, yellow}\}$
  $\mu(b) = \text{blue}, \mu(g) = \text{green},$
  $\mu(r) = \text{red}, \mu(t) = \text{table}, \mu(y) = \text{yellow}$
  $\mu(o) = \text{on}$, where on is defined
    by the following picture
\[ l_1[o(b, r)] = \mu(o)(l_1[b], l_1[r]) \\
= \text{on}(l_1[b], l_1[r]) \\
= \text{on}(\mu(b), \mu(r)) \\
= \text{on}(\text{blue, red}) \\
= T \text{ (true)} \]
Temporary convention

• For now, we will assume that each interpretation has:
  \[ \Delta = \{\text{blue, green, red, table, yellow}\} \]

  \[ \mu(b) = \text{blue}, \mu(g) = \text{green}, \]
  \[ \mu(r) = \text{red}, \mu(t) = \text{table}, \mu(y) = \text{yellow} \]

  but we may have a different \( \mu(o) \)
\[ l_2[o(b, r)] = \]
\[ l_2[o(g, y)] = \]
\[ l_2[o(g, y) \rightarrow o(b, r)] = \]
\[ l_2[\forall u \forall v \neg (o(u, v) \land o(v, u))] = \]

\[ l_2 \text{ is:} \]
\[ l_3[o(b, r)] = \]
\[ l_3[o(g, y)] = \]
\[ l_3[o(g, y) \rightarrow o(b, r)] = \]

\[ l_3[\forall u \forall v \neg(o(u, v) \land o(v, u))] = \]

\[ l_3 \text{ is:} \]
Closed Formula + Interpretation gives Truth Value

• What if formula is not closed, e.g.
  \( \neg (o(u, v) \land o(v, u)) \)

• If the formula is being considered in isolation, we close it by prepending \( \forall u \forall v \).

• The meaning is as if the variables were \( \forall u \forall v \) quantified.
Equality

• Equality = is a 2-ary predicate symbol.

• By convention, for every interpretation, \( \mu(=) \) is the **identity predicate** on the universe \( \Delta \).

• That is, \( \mu(=) = \{(\text{blue, blue}), (\text{green, green}), (\text{red, red}), (\text{table, table}), (\text{yellow, yellow})\} \)
$I_3[(o(r, x) \rightarrow (x = b))] =$

$I_3[(o(x, y) \rightarrow (x = g))] =$

$I_3$ is:
$l_3[\exists x \ o(r, \ x)] = l_3[(\exists x \ o(x, \ g)] = l_3$ is:
$I_4[\forall u \exists x o(x, u)] = I_4$ is:
$I_4[\forall u \exists x \ o(x, u) \lor u = b] =$

$I_4$ is:
\[ I_4[\forall u \exists x \ o(u, x)] = \]

\[ I_4 \text{ is:} \]
Devise Some **Universal** Formulas for Blocks Interpretations

- By universal, I mean a formula that is true in every blocks world interpretation
- You are allowed to add new predicates

- Example:

\[ \forall u \exists x \circ(u, x) \vee (u = t) \]

because every block is either on another block, or on the table.
More Examples

• $\forall x \neg o(x, x)$
New predicate \( a \)

- \( a(u, x) \) means \( u \) is “above” \( x \).
New predicate \( a \)

\( a(u, x) \) means \( u \) is “above” \( x \)

\( l_3[(a(r, b)] = T \quad l_3[(a(r, y)] = T \)
Now for blocks world or any other

• Devise some formulas that are true, regardless of the interpretation.

• These are called valid or universally valid.

• Example: \((\forall u \ o(u, u)) \rightarrow (\forall u \ \exists x \ o(u, x))\)