Regular Languages, Continued

March 28, 2011

CS 81: Computability and Logic

A Jewel of Theoretical Computer Science

The following are equivalent:

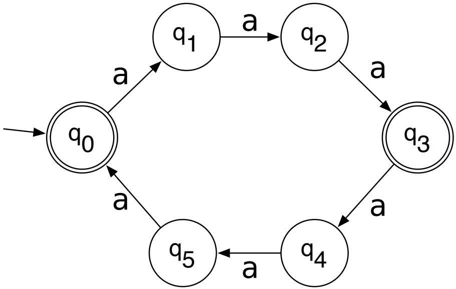
1. There is a DFA accepting the language L
2. [Rabin and Scott] There is an NFA accepting L
3. [Kleene] L is a regular set.

Completing the Equivalence: Automata to Regular Expressions

Two approaches:

1. Solving equations
2. Generalized NFAs

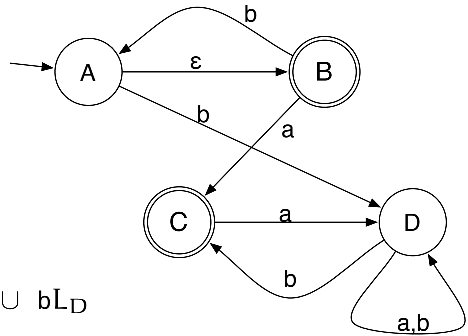
The Language of a State



Let L_q be the set of strings are accepted when starting from state q.

- ✓ What is $L_{q_0}, L_{q_1}, L_{q_2}, \ldots$?
- ✓ How is L_{q_1} related to L_{q_2} ?

Automaton as a System of Equations



- ✓ $L_A = \varepsilon L_B \ \cup \ bL_D$
- ✓ $L_B =$
- ✓ $L_C =$
- ✓ $L_D =$

Solving Equations using Arden's Rule

- ✓ The equation

$$L = AL \cup B$$

has the solution

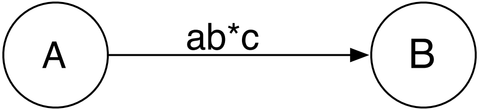
$$L = A^*B$$

- ✓ This is the smallest solution
  - If $\varepsilon \notin A$, the unique solution
  - Otherwise A^*C is a solution for any $B \subseteq C$.

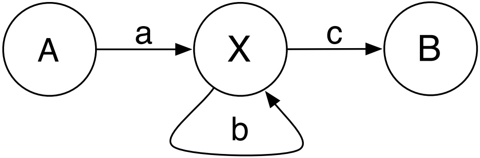
$$
\begin{aligned}
L_A &= L_B \cup \mathtt{b}L_D \\
L_B &= \varepsilon \cup \mathtt{b}L_A \cup \mathtt{a}L_C \\
L_C &= \varepsilon \cup \mathtt{a}L_D \\
L_D &= (\mathtt{a} \cup \mathtt{b})L_D \cup \mathtt{b}L_C
\end{aligned}
$$

Generalized NFAs

Just like an NFA, but edges have regular expressions rather than single symbols



Since regular expressions can be turned into NFAs, we aren't adding any extra power.

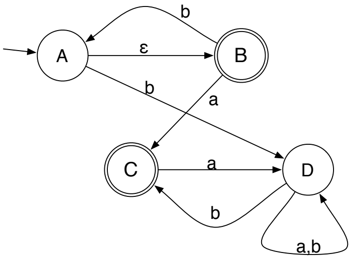


Regexp by Removing States

The strategy:

✓ Make sure our NFA has

- *One start state, with edges only going out*
- *One accept state, with edges only going in.*



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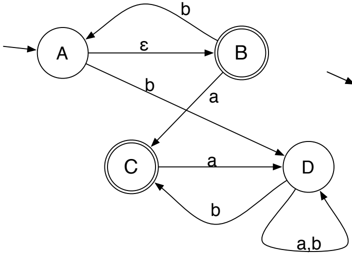

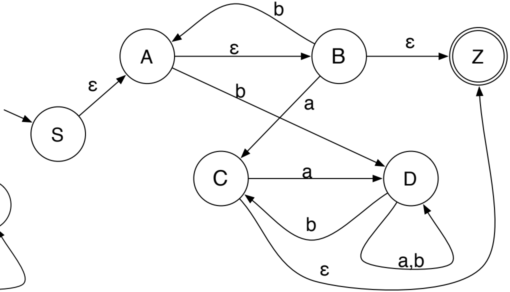

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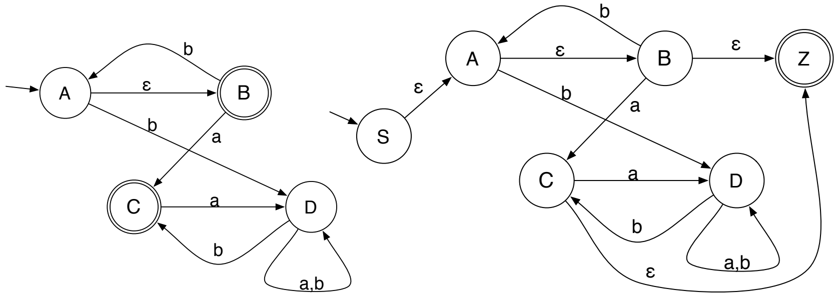
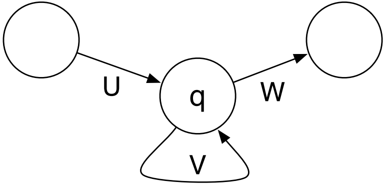


✓ Remove all the intermediate states (A–D), *one at a time.*

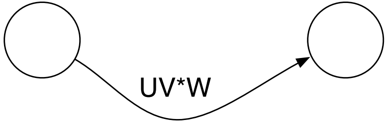
✓ In the end, we have one edge, labeled by our regexp.

Removing States

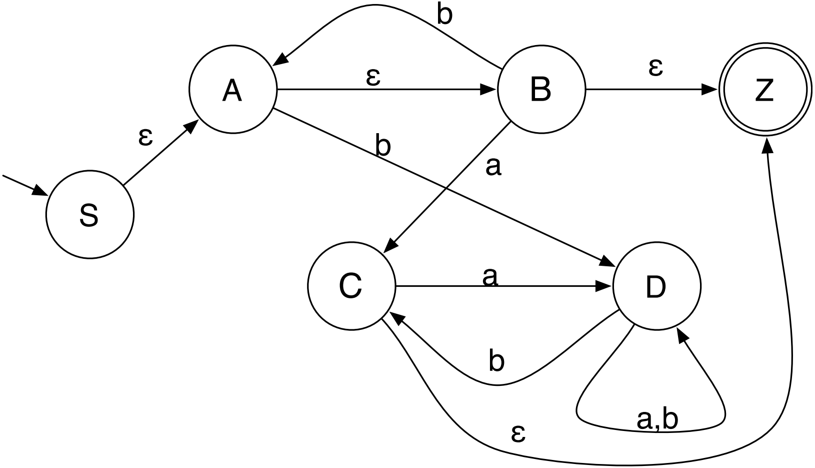
- ✓ When removing state **q**, replace every pair of in/out edges



by a single edge



Example



Regular Expression Review

- ✓ What are the usual "primitive" regular expressions?

- ✓ What other regular expressions have you seen?

```
M[ou]'?am+[ae]r .*([AEae]l[- ])?[GKQ]h?[aeu]+([dtz][dhz]?)+af[iy]
```

Give two strings matching this regular expression.

```
M[ou]'?am+[ae]r .*([AEae]l[- ])?[GKQ]h?[aeu]+([dtz][dhz]?)+af[iy]
```

Give two strings matching this regular expression.

```
Muammar Qaddafi
Mo'ammar Gadhafi
Muammar Kaddafi
Muammar Qadhafi
Moammar El Kadhafi
Muammar Gadafi
Mu'ammar al-Qadafi
Moamer El Kazzafi
Moamar al-Gaddafi
Mu'ammar Al Qathafi
Muammar Al Qathafi
Mo'ammar el-Gadhafi
Moamar El Kadhafi
Muammar al-Qadhafi
Mu'ammar al-Qadhdhafi
Mu'ammar Qadafi
Moamar Gaddafi
Mu'ammar Qadhdhafi
Muammar al-Khaddafi
Mu'amar al-Kadafi
Muammar Ghaddafy
Muammar Ghadafi
Muammar Ghaddafi
Muamar Kaddafi
Muammar Quathafi
Muammar Gheddafi
Muamar Al-Kaddafi
Moammar Khadafy
Moammar Qudhafi
Mu'ammar al-Qaddafi
Mu'ammar Muhammad Abu Minyar al-Qadhafi
```

From the RX library

Exercise

Give a regular expression for C identifiers:

- ✓ Can contain letters, digits, and underscores
- ✓ Must begin with a letter or underscore.
- ✓ E.g., `main` *or* `__Z6rotateiii`

Give a regular expression for Ada identifiers, which

- ✓ Can contain letters, digits and underscores
- ✓ Begin with a letter
- ✓ Have no consecutive underscores or an underscore at the end
- ✓ E.g., `woohoo32` *or* `Last_Nonzero_Row`

Exercise: Integer Constants in C

KERNIGHAN & RITCHIE
"THE C PROGRAMMING LANGUAGE"

A2.5.1 Integer Constants

An integer constant consisting of a sequence of digits is taken to be octal if it begins with `0` (digit zero), decimal otherwise. Octal constants do not contain the digits `8` or `9`. A sequence of digits preceded by `0x` or `0X` (digit zero) is taken to be a hexadecimal integer. The hexadecimal digits include `a` or `A` through `f` or `F` with values 10 through 15.

An integer constant may be suffixed by the letter `u` or `U`, to specify that it is unsigned. It may also be suffixed by the letter `l` or `L` to specify that it is long.

The type of an integer constant depends on its form, value and suffix. (See §A4 for a discussion of types.) If it is unsuffixed and decimal, it has the first of these types in which its value can be represented: `int`, `long int`, `unsigned long int`. If it is unsuffixed octal or hexadecimal, it has the first possible of these types: `int`, `unsigned int`, `long int`, `unsigned long int`. If it is suffixed by `u` or `U`, then `unsigned int`, `unsigned long int`. If it is suffixed by `l` or `L`, then `long int`, `unsigned long int`.

The elaboration of the types of integer constants goes considerably beyond the first edition, which merely caused large integer constants to be `long`. The `U` suffixes are new.

A2.5.2 Character Constants

Exercise: Integer Constants in C

Does your regular expression match:

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ 6

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ 6
- ✓ 9

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ 6
- ✓ 9
- ✓ 2u

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ 6
- ✓ 9
- ✓ 2u
- ✓ 2L

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ 6
- ✓ 9
- ✓ 2u
- ✓ 2L
- ✓ 2UL

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`
- ✓ `002`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`
- ✓ `002`
- ✓ `09`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`
- ✓ `002`
- ✓ `09`
- ✓ `02ul`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`
- ✓ `002`
- ✓ `09`
- ✓ `02ul`
- ✓ `0Xff90`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`
- ✓ `002`
- ✓ `09`
- ✓ `02ul`
- ✓ `0Xff90`
- ✓ `0x0`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`
- ✓ `002`
- ✓ `09`
- ✓ `02ul`
- ✓ `0Xff90`
- ✓ `0x0`
- ✓ `0x0F`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
- ✓ `2UL`
- ✓ `2lu`
- ✓ `2uu`
- ✓ `02`
- ✓ `002`
- ✓ `09`
- ✓ `02ul`
- ✓ `0Xff90`
- ✓ `0x0`
- ✓ `0x0F`
- ✓ `0x0FlU`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
- ✓ `9`
- ✓ `2u`
- ✓ `2L`
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- ✓ `002`
- ✓ `09`
- ✓ `02ul`
- ✓ `0Xff90`
- ✓ `0x0`
- ✓ `0x0F`
- ✓ `0x0FlU`
- ✓ `0x0x3`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
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- ✓ `0x0F`
- ✓ `0x0FlU`
- ✓ `0x0x3`
- ✓ `0`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
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- ✓ `02ul`
- ✓ `0Xff90`
- ✓ `0x0`
- ✓ `0x0F`
- ✓ `0x0FlU`
- ✓ `0x0x3`
- ✓ `0`
- ✓ `0lu`

Exercise: Integer Constants in C

Does your regular expression match:

- ✓ `6`
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- ✓ `09`
- ✓ `02ul`
- ✓ `0Xff90`
- ✓ `0x0`
- ✓ `0x0F`
- ✓ `0x0FlU`
- ✓ `0x0x3`
- ✓ `0`
- ✓ `0lu`
- ✓ `0x`

Closure Properties

A *family* of languages is a set of languages.

- ✓ The family of all finite languages
- ✓ The family of all languages
- ✓ The family of all regular languages

A family $\mathcal{F}$ is *closed under* an operation if applying the operation to languages in $\mathcal{F}$ always produces a result in $\mathcal{F}$.

Finite Languages

Is the family of finite languages closed under:

- ✓ Union? $(A \cup B)$
- ✓ Intersection $(A \cap B)$
- ✓ Concatenation? (AB)
- ✓ Star (A^*)
- ✓ Complement (A^c)

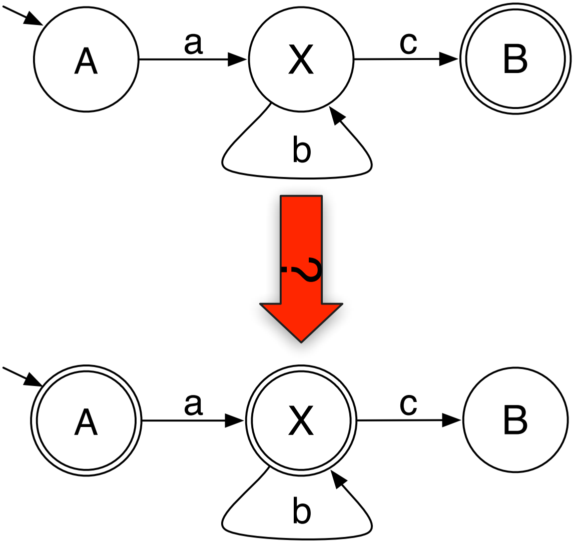
Regular Languages

The regular languages are closed under

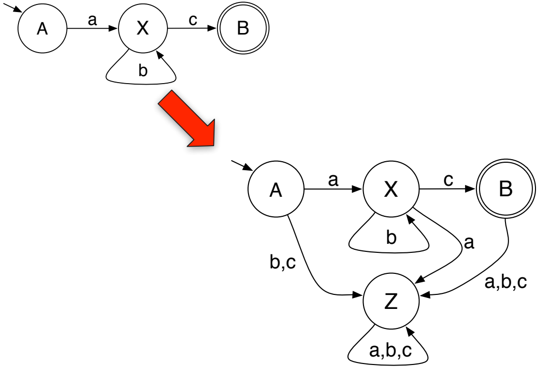
- ✓ Union? $(A \cup B)$
- ✓ Intersection $(A \cap B)$
- ✓ Concatenation? (AB)
- ✓ Star (A^*)
- ✓ Complement (A^c)

Proofs: Consider the corresponding automata...

Complement?



Complement!



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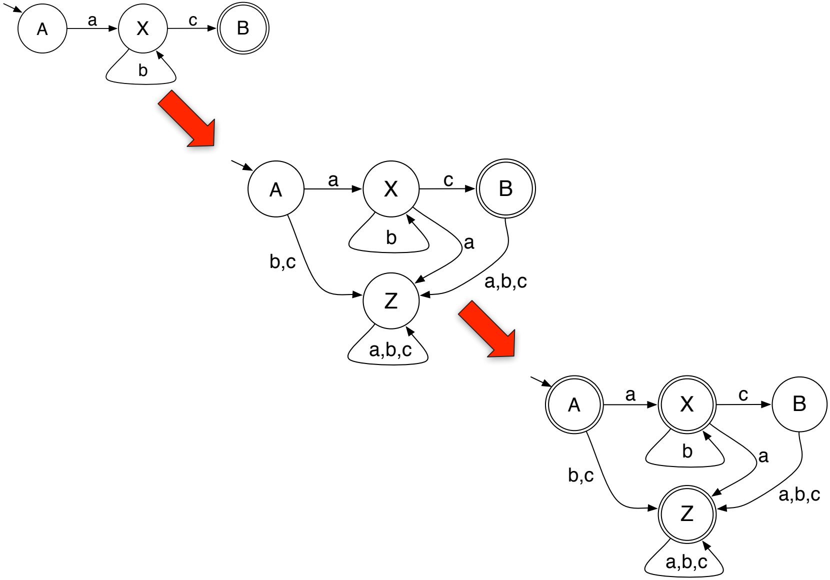

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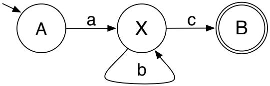
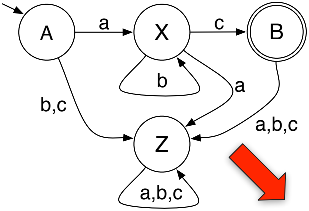
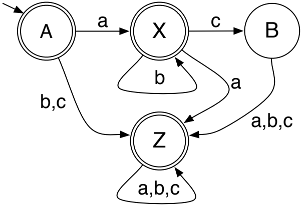


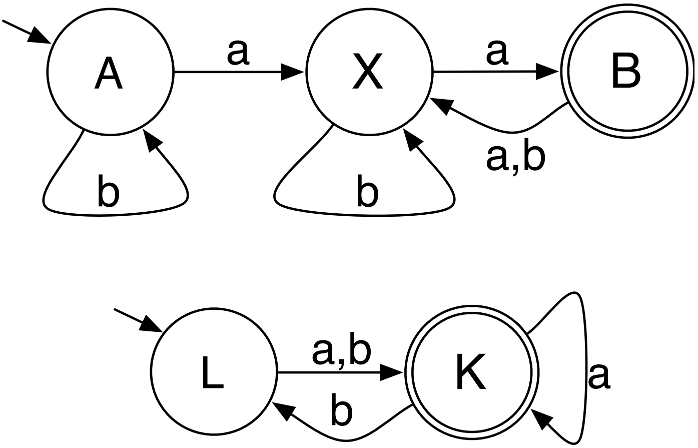




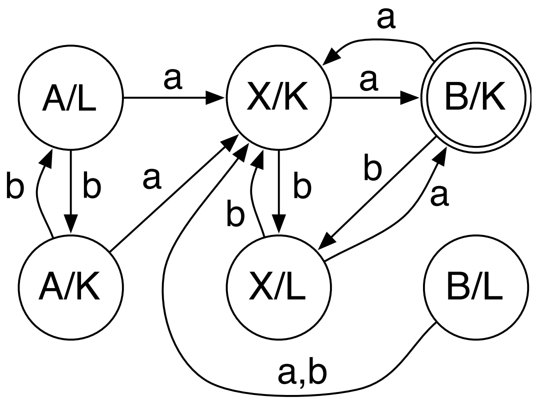
$$\text{DFA } M = (\Sigma, Q, \rightarrow, q_0, F)$$

$$\text{DFA } M^c = (\Sigma, Q, \rightarrow, q_0, Q \setminus F)$$

Intersection: Inputs



Intersection: Product Automaton



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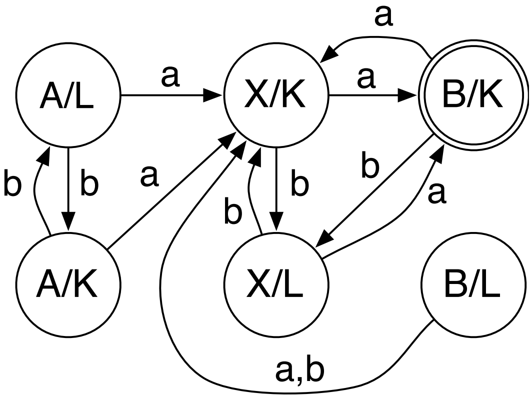

$$\text{DFA } M = (\Sigma, Q, \rightarrow, q_0, F)$$
$$\text{DFA } M' = (\Sigma, Q', \rightarrow', q_0', F')$$
$$\text{DFA } M \cap M' = (\Sigma, Q \times Q', \rightarrow_{\text{both}}, \langle q_0, q_0' \rangle, F \times F').$$