CS81 Assignment 5
Due Monday 3 March 2014 by 9 PM

Collaboration Policy: You are allowed to discuss problems with others. However, the work you turn in must be by your own hand, done without transcription from other sources. If you receive significant help, cite the problem and the nature and source of the help.

1. [5] In class, we described the structure of a soundness proof for classical natural deduction (propositional case). Prove that the RAA rule, in contextual form, is sound:

\[
\Gamma \cup \{\neg A\} \models \bot \\
\therefore \Gamma \models A
\]

(Assume the antecedent, then prove the consequent: Consider a valuation satisfying \( \Gamma \) ...)

2. [5] Do the same (as in 1) for \( \neg \)Introduction:

\[
\Gamma \models \bot \\
\therefore \Gamma \models \neg A
\]

2. [15] The proof of the completeness theorem given in the handout showed that every tautology is provable by constructing a natural deduction proof for it. By following the proof of the completeness theorem, what natural deduction proof would be constructed for:

\[
((p \rightarrow q) \rightarrow p) \rightarrow p
\]

3. [5] Let \( L(x, y) \) mean “\( x \) loves \( y \)”. Translate each of the following statements into a predicate logic formula. John, Mary, and Tom are constant symbols. Loving is not regarded as exclusive: If one person loves another, he/she might love others as well. By a “lover”, we mean a person who loves someone (including possibly him or herself).

   a. John loves Mary.
   b. Mary loves herself.
   c. John is a lover.
   d. John loves everyone except Mary.
   e. John loves exactly one person.
   f. Mary loves exactly two people.
   g. Tom loves no one.
   h. Tom loves everyone who does not love him.
   i. Tom loves everyone who does not love him/herself.
   j. Tom loves any and only those who do not love themselves.

Continuing with the nomenclature above, in 4-6, check each syllogism or sequent
for validity using the **tableau method**. If not valid, give a counterexample.

4. [10] Given
   a. Tom loves everyone who does not love him/herself (and maybe others).
   It follows that
   b. Tom loves himself.

5. [10] Given
   a. Everyone loves every lover.
   b. Mary loves herself.
   It follows that
   c. Mary loves John.

6. [10] (Tricky. Be careful) Given
   a. Tom loves any and only those who do not love him/herself.
   It follows that
   b. Tom loves Mary.

In 7-10, check each sequent for validity by the tableau method. If not valid, give a counterexample.

7. [10] \( \exists x (A(x) \rightarrow B(x)) \) \( \neg \) \( (\forall x A(x)) \rightarrow (\exists x B(x)) \)

8. [10] \( (\forall x A(x)) \rightarrow (\exists x B(x)) \rightarrow (\forall x (A(x) \rightarrow B(x))) \)

9. [10] \( \exists x \exists y (H(x,y) \lor H(y,x)), \neg \exists x H(x,x) \) \( \neg \) \( \exists x \exists y (x = y) \)

10. [10] \( \exists T, \forall x \exists y (P(x) \rightarrow Q(y)) \) \( \neg \) \( \exists y (\forall x (P(x) \rightarrow Q(y))) \)